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CANONICAL SURFACES WITH $p_g = p_a = 4$ AND $K^2 = 5, \ldots, 10$

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Introduction. Although many general results are now available in the theory of surfaces of general type, a complete classification is still lacking. As a matter of fact, for many classes of surfaces little is known about important properties, like, for instance, the behaviour of the canonical map; moreover, for a large number of classes even the existence is unknown. It is clear that the analysis of particular cases will give some insight in the general theory.

From this point of view we have tried to work out a systematic study of canonical regular surfaces with $p_g = 4$. This is the lowest value of p_g for which the canonical map can be birational. In this paper we expose our results. The main result (Theorem (8.1)) is that for any $n = 5, \ldots, 10$, there exists an algebraic family $\mathcal{K}(n)$ of surfaces in $\mathbf{P}^3$ such that the generic element in $\mathcal{K}(n)$ is a canonical surface with $p_g = p_a = 4$, $K^2 = n$, with ordinary singularities without exceptional curves of the first kind; moreover any such surface is in $\mathcal{K}(n)$; this family is irreducible, unirational of dimension $65 - 2n$. As a consequence we have the unirationality of the coarse moduli space of the surfaces of $\mathcal{K}(n)$. The techniques used to prove the theorem give also an explicit description of the above surfaces.

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