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BLASCHKE MANIFOLDS WITH TAUT GEODESICS

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A Blaschke manifold may be characterized as a Riemannian manifold whose tangent cut locus is a sphere at every point. Since every geodesic of a Blaschke manifold is a simply closed geodesic by 5.40 and 7.9 of [1], the Bott-Samelson theorem (7.23 of [1]) implies that a Blaschke manifold has the cohomology of a compact rank-one symmetric space. For a Blaschke manifold M, let M_1 denote the compact rank-one symmetric space having the same cohomology as M and normalized to have the same diameter as M. Then Blaschke's conjecture may be stated as follows:

Conjecture. *M is isometric to M_1.*

Marcel Berger has proved

THEOREM *A*. (Berger, D.1 of [1]) *If M_1 is a sphere or real projective space, then M is isometric to M_1.*

Since the geodesics of a Blaschke manifold are simply closed geodesics, they may be thought of as embedded circles. Given a point $p \in M$ and a geodesic γ of M, we define a function $L_p : \gamma \to \mathsf{R}$ by $L_p(x) = (d(x, p))^2$ for $x \in \gamma$ where d is the distance function of M.

Definition. A Blaschke manifold M has *taut geodesics* if for every geodesic γ, L_p has exactly two critical points for almost every $p \in M$.

Remark. Compact rank-one symmetric spaces are Blaschke manifolds with taut geodesics.

In this paper we prove

THEOREM *B*. *Let M be a Blaschke manifold with taut geodesics, then $Vol(M) \geqslant Vol(M_1)$ with equality holding if and only if M is isometric to M_1.*

Combining this result with Weinstein's theorem (2.21 in [1]), a Blaschke manifold with taut geodesics that is close to a compact rank one symmetric space is isometric to one. Thus, it would be possible to prove a local version of Blaschke's conjecture if the condition of having taut geodesics is satisfied by Blaschke manifolds close to compact rank one symmetric spaces. However, this may not be true since a circle that is tautly embedded may fail to be tautly embedded if the metric is slightly altered. In any case, aside from 5.81 of [1],

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