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KERGIN INTERPOLATION OF ENTIRE FUNCTIONS ON $\mathbf{C}^n$

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Introduction. Let $f(z)$ be an entire (analytic) function of one complex variable. The Newton series for f is the series

$$c_0 + c_1 z + c_2 z(z-1) + c_3 z(z-1)(z-2) + \cdots \tag{0.1}$$

where the coefficients c_j are chosen so that the value of the sum of the series matches the value of the function f for $z = 0, 1, 2, \ldots$. That is, for each integer m, the polynomial

$$p_m(z) = \sum_{j=0}^{m} c_j z(z-1) \cdots (z-(j-1))$$

satisfies

$$p_m(j) = f(j) \qquad \text{for} \quad j = 0, 1, \ldots, m. \tag{0.2}$$

Now $p_m(z)$ is a polynomial of degree $\leqslant m$ in z and is therefore the unique polynomial which satisfies (0.2). It is usually known as the Lagrange interpolating polynomial (see [7], [14] or [16]).

The question of the convergence of the Newton series or equivalently the question of convergence of the sequence of Lagrange interpolating polynomials has been studied extensively (see [4], [7] or [17]). One specific question is whether or not the series (0.1) converges uniformly on compact subsets of the plane to the function f.

Now, an entire function is not determined by its values on a discrete set of points. Thus, in general, one would not expect convergence unless there are some limits on the growth of the function f. A very interesting example is furnished by the function 2^z for which the series (0.1) does not converge uniformly on compact subsets of the plane. We will outline the proof of this. Since 2^j is an integer for $j = 0, 1, 2, \ldots$ it is easy to see that the coefficients c_j (which are divided differences of the function f—see §1.9) have the property that for each $j = 0, 1, 2, \ldots$ then $c_j j!$ is an integer. Now we proceed by contradiction. Assume 2^z can be represented in the form (0.1). Put $z = -1$. We obtain $1/2$ as a sum of integers which is impossible.

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