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# COBORDISMS OF EVEN-DIMENSIONAL LINKS

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**Introduction.** In this paper, we consider the question of whether or not every link of even dimension greater than or equal to four is cobordant to the trivial link. Using techniques developed by Cappell and Shaneson ([CS1], [CS2]), we provide a sufficient condition, dependent only upon the fundamental group, for an even-dimensional link to be null-cobordant.

Recall that a link of $m$ components, or $m$-link in the $n+2$-sphere $S^{n+2}$ is a smooth, oriented submanifold $\Sigma^n \subset S^{n+2}$, where $\Sigma^n = \Sigma_1 \cup \ldots \cup \Sigma_m$ is the ordered, disjoint union of $m$ manifolds that are $PL$-homeomorphic to $S^n$. Two $m$-links $\Sigma_i = \Sigma_{i,1} \cup \ldots \cup \Sigma_{i,m} \subset S^{n+2}$, $i = 1,2$, are said to be *cobordant* if there is a smooth, oriented submanifold $V$ $S^{n+2} \times [0,1]$, $PL$-homeomorphic to $\Sigma_0 \times [0,1]$, which meets the boundary transversely in $V$ so that $V \cap (S^{n+2} \times i) = \Sigma_i$ for $i = 0, 1$.

A link is *trivial* if the components bound disjoint discs inside $S^{n+2}$. A *null-cobordant* link is one that is cobordant to a trivial link. Finally, a *boundary link* is one in which the components bound disjoint $n+1$-dimensional submanifolds of $S^{n+2}$.

The question above is a special case of the question of whether or not every link is cobordant to a boundary link, since it is known by [G1] that every even-dimensional boundary link is null-cobordant. Odd-dimensional links are discussed in detail in my thesis, [D], on which this paper is based.

We will make a few definitions, then state our main result. A *link group* is one that is realized as $\pi_1(S^{n+2} - \Sigma^n)$ for some $\Sigma^n \subset S^{n+2}$, $n \geqslant 3$. By a theorem of Kervaire ([K]; see also [R], pp. 353–54), generalized to links, $\pi$ is an *m-link group* if and only if $\pi$ is finitely presented, $H_1(\pi; Z) = Z^m$, $H_2(\pi; Z) = 0$, and $\pi$ has weight $m$, i.e., $\pi/\langle x_1, \ldots, x_m\rangle = 1$, for some subset $\{x_1, \ldots, x_m\}$, where "$\langle\ \rangle$" means "normal closure." From now on, we will assume that all our links have $m$ components, and all our link groups have weight $m$, for some fixed integer $m \geqslant 2$.

Let $G$ be a group. We will say that $G$ is *n-aspherical* if $H_i(G; Z) = 0$ for $2 \leqslant i \leqslant n$. For example, all link groups are 2-aspherical.

THEOREM. *Let $k \geqslant 2$, and let $\Sigma^{2k} \subset S^{2k+2}$ be a link. Suppose there is a map $\theta : \pi_1(S^{2k+2} - \Sigma^{2k}) \to \pi$ to a $2k+2$-aspherical link group such that $\langle \text{image}\,\theta \rangle = \pi$. Then $\Sigma^{2k} \subset S^{2k+2}$ is null-cobordant.*

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