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KERNELS FOR THE LOCAL SOLVABILITY OF THE TANGENTIAL CAUCHY–RIEMANN EQUATIONS

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1. Introduction. In 1972, Andreotti and Hill [AH] showed that if M is a real hypersurface in $\mathbf{C}^n$ with certain convexity properties, then its local $\bar{\partial}_M$-cohomology vanishes in certain bidegrees. Since that time, mathematicians have sought explicit kernels to represent a local solution to the tangential Cauchy–Riemann equations, in much the same way that the Cauchy kernel $1/\pi z$ represents a solution to the equation $(\partial g/\partial \bar{z}) = f, f \in C_0^\infty(\mathbf{C})$. In 1977, G. Henkin [$H_3$] accomplished this in the case M is strictly pseudoconvex. In this paper, we find an explicit kernel which represents a local solution to the equation $\bar{\partial}_M g = f$, where f is a smooth form on M of bidegree (r,s), provided the Levi form of M has at least $\max\{s+1, n-s\}$ eigenvalues of the same sign. Our convexity assumption is the same as the one assumed in [AH] and it is slightly stronger than the $Y(s)$ condition assumed in Folland and Kohn [FK] for the global solvability of $\bar{\partial}_M$. In the case M is strictly pseudoconvex, our solution agrees with Henkin's. Our kernel approach also exhibits the possible obstructions to locally solving the tangential Cauchy-Riemann equations in the bidegrees where the local $\bar{\partial}_M$— cohomology does not a-priori vanish.

In our work, we employ a general class of kernels which was first introduced by Henkin, Romanov, and Skoda and then generalized and streamlined by Harvey and Polking [HP]. Much of our work involves constructing a new local support function for these kernels.

We have organized our work as follows. After a short preliminary chapter, we present a general approach to the tangential Cauchy–Riemann equations. The concepts in this chapter are presented in Harvey-Polking [HP] and Henkin [H_3] in the strictly pseudoconvex case, and there is no new work involved in

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