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# A REFLECTION PRINCIPLE FOR DEGENERATE REAL HYPERSURFACES

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**Introduction.** This paper is concerned with the continuation of a biholomorphic mapping $f$ across an analytic real hypersurface in $\mathbf{C}^n$ by means of a reflection principle. In particular, suppose that $f$ maps $D$ to $D'$, where these are two bounded domains with smooth real analytic boundaries. Then we show

THEOREM. *Associated to $D$ are an integer $k$ and a real analytic set $A \subset \partial D$, $\dim A \leqslant 2n-3$, such that if $f$ and $f^{-1}$ are $C^k$ up to the boundary then $f$ continues holomorphically past $\partial D \backslash A$.*

More precise results are stated below in theorem (3.1) and its corollary (3.2). Reflection principles for strongly pseudoconvex hypersurfaces were given in [7], [9], [10], and [11]. We extend the method here to more general hypersurfaces at the expense of assuming more initial boundary regularity. However, if the domains $D$ and $D'$ are also assumed to be pseudoconvex then $f$ is known to be smooth up to the boundary [1], [14].

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**1. Invariant complex varieties.** Let $M$ denote an analytic real hypersurface in $\mathbf{C}^n$. We shall use both $z = (z^1, \ldots, z^n)$ and $w = (w^1, \ldots, w^n)$ to denote coordinates of points in $\mathbf{C}^n$. For a fixed point $z_0$ in $M$ we assume the coordinates are chosen so that $z_0 = 0$ and the real tangent plane to $M$ at $z_0$ is given by $Im z^n = 0$. Choose a real analytic defining function $r = r(z, \bar{z})$ for $M$ near $z_0$ and a polycylindrical neighborhood $U_0 = \{z : |z^i| < \rho_0\}$, such that $r(z, \overline{w})$ converges for $(z, w) \in U_0 \times U_0$, and $r_n = \partial r(z, \overline{w})/\partial z^n \neq 0$.

For each $w$ in $U_0$ define

$$Q_w \equiv U_0 \cap Q_w = \{z \in U_0 : r(z, \overline{w}) = 0\}, \tag{1.1}$$

$$A_w \equiv U_0 \cap A_w = \{z \in U_0 : Q_z = Q_w\}. \tag{1.2}$$

Clearly, $z \in A_z$, and $w \in A_z$ if and only if $A_w = A_z$. By the reality condition on $r$,

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