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LOCALLY ISOMETRIC LIFTINGS FROM QUOTIENT C^*-ALGEBRAS

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§1. **Introduction.** A linear map ϕ between C^*-algebras A and B is said to be completely positive if the associated maps

$$\varphi \otimes I_m : A \otimes M_m \to B \otimes M_m, \quad m \geqslant 1$$

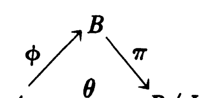
are all positive. This notion was introduced by Stinespring in [10], where a powerful representation theorem for such maps was obtained. Building upon this, Arveson [2, 3] undertook a detailed study of completely positive maps and their implications for operator algebras. Among other things Arveson connected the notions of matrix range inclusion and completely contractive maps in the following theorem.

THEOREM 1.1. (*Arveson*). *Let S and T be Hilbert space operators (perhaps acting on different spaces). Then the following conditions are equivalent.*

(i) $W_n(S) \subseteq W_n(T)$ *for every* $n \geqslant 1$.

(ii) $\|A \otimes I + B \otimes S\| \leqslant \|A \otimes I + B \otimes T\|$ *for every pair* A, B, *of* $n \otimes n$ *matrices and every* $n \geqslant 1$.

In another direction, Choi and Effros [7] have shown that if A and B are C^*-algebras, J a closed two-sided ideal in B and if θ is a nuclear map from $A \to B/J$, then there exists a map ϕ for which the following diagram commutes:



Moreover if $\theta(e) = e$, ϕ may be chosen with the same property. Applying this result to the case when $A = B/J$, B is nuclear and ϕ is the identity map yields a completely positive map $\phi : B/J \to B$ for which $\pi\phi = \theta$. Clearly $\|\phi(a)\| \geqslant \|a\|$ for all $a \in B/J$. However, since $\phi(e) = e$, ϕ is easily seen to be completely isometric. From the equivalence of conditions (i) and (ii) above it follows that $W_n(\phi(T)) = W_{n,e}(T)$ for all n.

It was the hope of these authors that a local version of the above argument might hold true for *all* C^*-algebras. For a given $T \in B$, if one could find a $j \in J$ such that $W_n(T + j) = W_{n,e}(T)$ for all n this would imply the existence of a map $\phi : B/J \to B$ for which $\phi|_{\mathrm{sp}\{e,T,T^*\}}$ was completely isometric. As yet, these authors have been unable to obtain this result. Nevertheless the following is true.

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