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A PROOF OF GEHRING'S LINKED SPHERES CONJECTURE

MICHAEL E. GAGE

1. Introduction. In this paper we prove the following theorem:

THEOREM 1.1. *If* M^k *and* N^l *are* k *and* l *spheres which are differentiably imbedded in* E^{k+l+1} *such that they link and such that* $\operatorname{dist}(M^k, N^l) \geqslant 1$ *then* $\mathcal{H}^k(M^k) \geqslant \omega_k$ *and* $\mathcal{H}^l(N^l) \geqslant \omega_l$.

Here $\mathcal{H}^k$ refers to k dimensional Hausdorff measure, ω_k is the k dimensional measure of the unit k sphere, and $\operatorname{dist}(M^k, N^l) = \inf\{\operatorname{dist}(x, y) \mid x \in M^k$ and $y \in N^l\}$.

In the case $k = l = 1$ the theorem was conjectured by F. W. Gehring [8] and proved by M. Edelstein and B. Schwarz [3] and by M. Ortel [14]. Gehring [7] showed the existence of some positive lower bound for all k and l. Using a stronger linking assumption and minimal surface theory R. Osserman [13] showed that if $k = 1$ and $l \geqslant 1$ then one can prove that the curve has length greater than 2π. In [6] the author proved the theorem for the case $k = 1$, $l = 2$ when the linking number of the two spheres is not zero.

Recently E. Bombieri and L. Simon [1] have proved that

THEOREM 1.2 (Bombieri-Simon). *If the linking number of* M^k *and* N^l *is not zero and* $\operatorname{dist}(M^k, N^l) \geqslant 1$ *then* $\mathcal{H}^k(M^k) \geqslant \omega_k$ *and* $\mathcal{H}^l(N^l) \geqslant \omega_l$.

Theorem 1.1 extends this to the general case. (See section 2 for definitions of linking number and linked.)

Proof of theorem 1.1. *Case* 1. If $l = k = 1$ the theorem is due to M. Edelstein, B. Schwarz and M. Ortel.

Case 2. If $l \geqslant k \geqslant 2$ then the linking number is not zero if and only if the two spheres are linked. The proof of this follows directly from results of A. Haefliger and H. Whitney and is sketched in section 3. The theorem is now proved using theorem 1.2.

Case 3. If $l > k = 1$ then the theorem follows from theorem 4.5 in this paper.

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