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COVARIANT REPRESENTATIONS ON THE CALKIN ALGEBRA II

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To my father, on his 65th birthday.

1. Introduction. Let G be a separable metrizable compact group and suppose given a C^*-algebra A and a homomorphism $G \to \text{Aut}(A)$ written $g \rightsquigarrow a^g$ (with $a^{gh} = (a^h)^g$). Define

$$A_G = \{a \in A \mid \text{the function } g \mapsto a^g \text{ is continuous}\}.$$

Then A_G is a C^*-algebra. The examples of prime interest to us arise as follows. Let $\lambda : G \to \mathcal{L} = \mathcal{L}(L^2(G, l_2))$ denote the left regular representation of G with infinite multiplicity. Let $T^g = \lambda_g T \lambda_g^*$. The resulting C^*-algebra $\mathcal{L}_G$ contains the compact operators $\mathcal{K}$. It is smaller than $\mathcal{L}$ (unless G is finite). Similarly, G acts on the Calkin algebra $\mathcal{Q} = \mathcal{L}/\mathcal{K}$ and one obtains $\mathcal{Q}_G$. It is evident that $\mathcal{L}_G/\mathcal{K} \subseteq \mathcal{Q}_G$.

In [7] various general properties of $\mathcal{L}_G$ and $\mathcal{Q}_G$ were found. The present paper is devoted to extending those results and to exploring other related topics.

To be more specific, section 2 is devoted to lifting theorems. When can some $a \in \mathcal{Q}_G$ with specified properties be lifted to some $A \in \mathcal{L}_G$ with the same properties? Section 3 is concerned with G-automorphisms and centralizers. In particular, we show that the centralizer of $\mathcal{Q}_G$ in $\mathcal{Q}$ is trivial and that G-automorphisms of $\mathcal{L}_G$ and $\mathcal{Q}_G$ are inner whenever there is a realistic hope that they could be. In section 4, working under the added assumption that G is compact abelian, we prove that $\mathcal{L}_G$ is isomorphic to the crossed product algebra of the dual group of G acting upon the fixed elements of $\mathcal{L}_G$. This yields information about the invariant ideals in $\mathcal{L}_G$.

The C^*-algebra $\mathcal{L}_G$ has appeared as the norm-continuous part of a C^*-G-system, in analogy with von Neumann algebra systems; a great deal is known (when G is abelian) in that setting [6, 9, 11]. In particular, Olesen and Pedersen have generalized our theorems in section 4 to locally compact abelian groups [10].

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