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COMPLETION OF LINK MODULES

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1. Introduction. Let L be a tame link of μ ($\mu > 1$) components imbedded in the 3-sphere, S^3. One of the most important invariants of such a link is what R. Crowell [7] calls the *link module sequence*:

$$0 \to B \to A \to I(H) \to 0. \tag{1}$$

This is a short exact sequence of $Z(H)$-modules, where H denotes the abelianized fundamental group of $S^3 - L$, $I = I(H)$ is the augmentation ideal in the integral group ring $Z(H)$ and A is the Alexander module. (A detailed exposition of the link module sequence is given in §4 of the present paper.) The main purpose of this paper is to study the associated short exact sequence

$$0 \to \hat{B} \to \hat{A} \to \widehat{I(H)} \to 0 \tag{2}$$

consisting of the I-adic completions of the original modules (the definition of the I-adic completion of a module is given in books on commutative algebra; see [13], Chap. VIII for example). This is an exact sequence of modules over the ring $\widehat{Z(H)}$, which is the I-adic completion of $Z(H)$.

There are two distinct reasons for introducing and studying the modules in the exact sequence (2):

(a) The modules in the sequence (2) are invariant under isotopy, cobordism, concordance, and I-equivalence of links. This is *not* true of the modules in (1). Various corollaries follow; for example, the ideal in the ring $\widehat{Z(H)}$ generated by the Alexander polynomial is also an invariant of isotopy, cobordisms, etc.

(b) The so-called "Chen groups" [10] of the link L are determined by the module $\hat{B}$ together with its I-adic filtration. In fact, this seems to be the most reasonable way to try to determine the structure of the Chen groups. The Chen groups can be thought of as a first approximation to the lower central series quotients of the group $G = \pi_1(S^3 - L)$. (The Chen groups of L are the lower central series quotients of G/G''.) However, the Chen groups are much more amenable to computation than the lower central series quotients.

2. Statement of results. We will consistently denote the kth lower central series subgroup of a group G by $\Gamma_k(G)$. To be precise, $\Gamma_1(G) = G$ and $\Gamma_{k+1}(G) = [\Gamma_k(G), G]$ for all $k \geqq 1$. This paper originated in the author's attempts to determine the structure of the lower central series quotient groups

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