Vol. 47, No. 2 DUKE MATHEMATICAL JOURNAL © June 1980

THE Γ-IDEAL AND SPECIAL ZETA-VALUES

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Introduction. In [2], we presented an exposition of L. Carlitz's von-Staudt theorem for $\mathsf{F}_r[T]$ ($r = p^n$, p a rational prime), which determines the denominator of certain Bernoulli-style numbers. This seminal work contains the beginnings of many fruitful themes. Among them are: The study of new types of zeta-values, factorials and Hurwitz series for $\mathsf{F}_r[T]$, and Γ-functions for $\mathsf{F}_r[T]$. It is our purpose here to place these results in their most natural and general setting, where we replace $\mathsf{F}_r[T]$ with the general rings A of [4]; i.e., A is the ring of functions holomorphic away from ∞, a rational point, on a smooth, projective, geometrically irreducible curve, C, over F_r. We set k to be the function field and K its completion at ∞.

As A need not be a p.i.d., individual special-values alone do not suffice and one must consider the *ideals* generated by many special-values. So, we have "idealized" zeta-values, Γ-ideals, etc. For instance, to get an appropriate Γ-ideal, we need to view Carlitz's factorial *as a function* and look at the ideal generated by its values. In consequence of the passing to ideals, the proof here is more complicated than that of [2].

In the course of this work, some very important general principles became clear. The first is what we call the "two-variable principle": Functions such as L-series or Γ-functions, whether considered at the prime ∞ or at finite primes by interpolation, are natural continuous functions on a product $X \times Y$, where Y is Z_p or $\mathsf{Z}_p \times$ {a finite abelian group} and X is a characteristic $-p$ space. Further, if $y \in Y$ is fixed, the resulting function on X is given by a power series with a

Received November 10, 1979. Revision received March 5, 1980.