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REPRESENTATION THEORY OF THE HILBERT-LIE GROUP $U(\mathfrak{H})_2$

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Let $\mathfrak{k}$ denote the Lie algebra of all skew-adjoint Hilbert-Schmidt operators on a separable Hilbert space $\mathfrak{H}$. We let $U(\mathfrak{H})_2$ be the corresponding (linear) Hilbert-Lie group of all unitary operators U on $\mathfrak{H}$ satisfying the congruence:

$$U \equiv I \pmod{\mathcal{C}_2},$$

where $\mathcal{C}_2$ denotes the space of all Hilbert-Schmidt operators.

The purpose of this paper is to study the representation theory of $K = U(\mathfrak{H})_2$ by means of the orbit method. We summarize our results. In 1.2, we single out a special class $W_{\pm}(\mathfrak{k})$ of linear functionals on $\mathfrak{k}$, such that the coadjoint orbit $\mathcal{O}_f$, $f \in W_{\pm}(\mathfrak{k})$, admits a K-invariant complex structure (theorem 1.4) and if is the differential of a character λ_f of K_f. Hence, K acts holomorphically on the holomorphic sections on the homogeneous line bundle E_f over K/K_f determined by λ_f. We introduce a K-invariant inner product on the space of tame sections (2.2 and theorem 2.4) by using the integration theory on approximating finite-dimensional unitary subgroups. Existing integration theory does not readily apply to functions on K/K_f. The behavior of the transition functions on K/K_f precludes Gaussian integration on manifolds (see, for references, H. H. Kuo [1]), while the integration algebras given by Shale [2] do not contain any continuous functions on K/K_f. Our procedures allow us to form the holomorphically induced representation Ind(f, hol) which is irreducible (theorem 2.7). As a substitute for the left regular representation, we introduce the holomorphic Fock space (section 3). In decomposing Fock space into irreducibles, we obtain a canonical equivalence between Ind(f, hol) and a maximal symmetry class of tensors. In section 4, the norm-continuous representations of $U(\mathfrak{H})_2$ are classified by a doubly-indexed signature. These representations are GCR-representations of the smaller group $U(\infty)$ (theorem 4.5). $U(\mathfrak{H})_2$ is not a type I group; in fact, large families of positive-definite functions on $U(\infty)$ which correspond to primary finite representations given by Voiculescu [1] extend to $U(\mathfrak{H})_2$ (theorem 5.2). Certain quasi-free states of the CAR give rise to type III primary representations of $U(\mathfrak{H})_2$ (theorem 5.5).

The group $U(\mathfrak{H})_2$ was explicitly introduced by Shale [1] in studying the symmetries of the free boson field. The representations Ind(f, hol) are equivalent

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