

ON THE VARIETY OF SPECIAL LINEAR SYSTEMS ON A GENERAL ALGEBRAIC CURVE

PHILLIP GRIFFITHS AND JOSEPH HARRIS

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0. Introduction

(a) On a smooth algebraic curve C of genus g we consider a divisor D of degree d . A classical problem is to determine the dimension $h^0(D)$ of the vector space $H^0(D)$ of rational functions having poles only on D , or equivalently the dimension $r(D)$ of the complete linear system $|D| = \mathbf{P}(H^0(D))$ of effective divisors linearly equivalent to D . Denoting by K the canonical divisor, the Riemann-Roch formula

$$r(D) = d - g + h^0(K - D)$$

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