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MELLIN TRANSFORMS AND SCATTERING THEORY I. SHORT RANGE POTENTIALS

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§1. **Introduction.** In [5] E. Mourre proved asymptotic completeness for a class of Hamiltonians $H = -\frac{1}{2}\Delta + V$ by proving compactness of the operators $(\Omega^{\mp} - 1)g(H_0)P_{\pm}$. Here $g \in C_0^\infty(0, \infty)$ and P_-, P_+ are projections onto "incoming" and "outgoing" subspaces of $\mathcal{H} = L^2(\mathbb{R}^n, d^n x)$ under the free evolution, in a sense made precise below. Mourre's argument introduces at least two simplifications of the work of V. Enss [2]: (1) the replacement of Enss's phase space decomposition by the partition of unity $1 = P_+ + P_-$ and (2) the use of compactness arguments rather than an "Enss Decomposition Principle" (see [8]).

The operators P_+ (resp. P_-) project onto the positive (resp. negative) spectral subspaces of the operator $D = \frac{1}{2}(\mathbf{x} \cdot \mathbf{p} + \mathbf{p} \cdot \mathbf{x})$ which generates dilations. Intuitively one expects that, under the free evolution $e^{-itp^2/2}$, vectors in $\mathcal{H}_+ = P_+ \mathcal{H}$ should escape to ∞ as $t \to +\infty$ and vectors in $\mathcal{H}_- = P_- \mathcal{H}$ should escape to ∞ as $t \to -\infty$. Below we study the operators $e^{-itp^2/2}g(H_0)P_{\pm}$ using the Mellin transform [9] and show that this is the case. The resulting estimates are then used to follow the compactness and completeness arguments of [5].

To state our result we assume the following hypothesis on the pair of self-adjoint operators $(H, H_0) : H_0 = -\frac{1}{2}\Delta$ and $V = H - H_0$ so that

(i) H is self-adjoint and $(H + i)^{-1} - (H_0 + i)^{-1} \in \mathcal{J}_\infty$, the ideal of compact operators, and

(ii) there exist integers $\alpha, \beta \geqslant 1$ so that the bounded, monotone decreasing function h given by $h(R) = \|(H + i)^{-\alpha}V(H_0 + i)^{-\beta}F(|x| \geqslant R)\|$ is in $L^1((0, \infty), dR)$.

(Here and elsewhere $F(x \in S)$ is the projection

$$F(x \in S)f(x) = \begin{cases} f(x) & \text{if } x \in S \\ 0 & \text{otherwise} \end{cases}$$

for Borel sets S.)

In condition (ii) the formal expression $(H + i)^{-\alpha}V(H_0 + i)^{-\beta}$ is understood to mean the difference $(H + i)^{-\alpha+1}(H_0 + i)^{-\beta} - (H + i)^{-\alpha}(H_0 + i)^{-\beta+1}$, which is

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