Vol. 47, No. 1 DUKE MATHEMATICAL JOURNAL © March 1980

ON ENSS' APPROACH TO SCATTERING THEORY

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1. Introduction. We re-work Enss' approach to two-body, non-relativistic, short range scattering with a number of minor modifications, which are mostly of expository significance. (i) By a simple re-ordering of the proof we eliminate the use of Wiener's theory or the RAGE theorem. (ii) By generalizing Cook's lemma and extending the definition of an "Enss' potential" we are able to treat more singular Hamiltonians than was previously possible. (iii) We give another proof of the Enss decomposition principle which relies on a continuous decomposition of phase space. (iv) We give an analysis of the concept of mutual subordination, which is also of importance in the Kato–Birman approach to scattering theory. In all major respects our exposition follows [3], [9] or [12]; an entirely different proof of a result very similar to our Theorem 4 has recently been given by Perry [7].

We investigate the scattering between the self-adjoint operators H and H_0 on $L^2(\mathsf{R}^d)$, and for definiteness assume that

$$H_0 = -\tfrac{1}{2}\Delta$$

although many other free Hamiltonians can be treated by the same method [12]. We write

$$V = H - H_0$$

so that V is a bounded quadratic form on $\mathrm{Dom}(H) \times \mathrm{Dom}(H_0)$ with the usual norms. Alternatively V is a bounded linear operator from $\mathrm{Dom}(H_0)$ to $\mathrm{Dom}(H)^*$. We do not assume the existence of V as an operator on $\mathcal{H}$.

Let $\mathfrak{S}$ be the class of continuous functions $F : \mathsf{R} \to \mathsf{C}$, such that $|F(x)| \geqslant 1$ for all x and $|F(x)| \to \infty$ as $x \to \pm\infty$.

HYPOTHESIS 1. *The pair H, H_0 are mutually subordinate in the sense that*

$$\|F(H)G(H_0)^{-1}\| + \|F(H_0)G(H)^{-1}\| < \infty$$

for certain $F, G \in \mathfrak{S}$.

According to Theorem 13 we may alternatively assume that

$$(H+i)^{-1} - (H_0+i)^{-1}$$

is compact. Compare [7].

Received November 7, 1979.