Vol. 47, No. 1 DUKE MATHEMATICAL JOURNAL © March 1980

ON A CLASS OF PSEUDO-DIFFERENTIAL OPERATORS WITH DOUBLE CHARACTERISTICS

PAUL GODIN

1. Introduction. In this paper, we study the singularities of solutions of pseudo-differential equations corresponding to some pseudo-differential operators which have a real principal symbol with possibly critical points.

Our starting point has been the differential operator

$$L_k = D_1^2 - x_1^{2k+1} D_2^2$$

where $D_j = (1/i)\partial/\partial x_j$ for $j = 1, 2$; $i^2 = -1$, $(x_1, x_2) \in \mathsf{R}^2$ and k is a positive integer.

L_0 is just the well known Tricomi operator which is of real principal type in the sense of [2]. Hence theorem 6.1.1. of [2] tells us that L_0 propagates the singularities along its null bicharacteristic strips.

Denote by (ξ_1, ξ_2) the dual variable of (x_1, x_2). Let ρ be a point of $T^*\mathsf{R}^2\backslash 0$ where x_1 and ξ_1 vanish. If $k > 0$, L_k is not of real principal type at ρ since dL_k vanishes at that point. Let u be a Schwartz distribution in R^2. If $L_k u$ is regular in a conic neighbourhood of ρ and if u is singular at ρ, what can be said about the singularities of u close to ρ? To study this problem we shall reason as follows. First, it is not hard to see that there are exactly two null bicharacteristic strips of L_k, say ν^+ and ν^-, contained in the region $x_1 > 0$ and tending to ρ. By theorem 6.1.1. of [2], the singularities of u propagate along ν^+ and ν^- since L_k is of real principal type there. From this, one can easily deduce that if $L_k u$ is regular in a conic neighbourhood of ρ and if u has two points of regularity sufficiently close to ρ, one on ν^+ and one on ν^-, the only possible singularities of u close to ρ are contained in the set where $x_1 = \xi_1 = 0$. Suitable estimates can be used to show that, in fact, there are no singularities of u close to ρ. Therefore, if $L_k u$ is regular close to ρ and u is singular at ρ, either ν^+ or ν^- is contained in the set of singularities of u in a neighbourhood of ρ.

In this paper, we are going to apply the idea we have just described to a class of at most doubly characteristic pseudo-differential operators which contains L_k.

We shall have to:

(i) Prove suitable estimates.

(ii) Study the null bicharacteristic strips near the critical points of the principal symbol.

(iii) Combine (i) and (ii) to deduce information about the propagation of singularities.

Received August 27, 1979. Revision received November 13, 1979.