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REAL STRUCTURE IN THE HYPERFINITE FACTOR

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Introduction. Among the finite factors the hyperfinite one is of main importance. A. Connes has in a series of papers [1, 2, 3] done penetrating work on this von Neumann algebra, so it is by now very well understood, even though smaller questions remain to be answered. One of them, which he asked me to look into, is how many conjugacy classes of involutory *-anti-automorphisms there are on the hyperfinite factor. Since $B(H)$—the bounded operators on a complex Hilbert space H—has two conjugacy classes of involutory *-anti-automorphisms, as follows from the classification of irreducible weakly closed Jordan algebras of self-adjoint operators on H, see [7, 8], the same might be expected for the hyperfinite factor. However, it will be shown in the present paper that the two classes collapse into one in the hyperfinite factor, so there is only one conjugacy class.

The proof of the above result consists of showing that there is up to conjugacy a unique real von Neumann algebra which generates the hyperfinite factor. Let M denote the hyperfinite factor of type II_1, and let α be an involutory *-anti-automorphism of M. Let $\mathfrak{R} = \{x \in M : \alpha(x) = x^*\}$. Then $\mathfrak{R}$ is a real von Neumann algebra, i.e., $\mathfrak{R}$ is a *-algebra over the reals which is weakly closed and satisfies $\mathfrak{R} \cap i\mathfrak{R} = \{0\}$ and $\mathfrak{R} + i\mathfrak{R} = M$. The proof consists of showing that there is an increasing sequence of real subalgebras $\mathfrak{R}_n$ of $\mathfrak{R}$, with $\mathfrak{R}_n$ isomorphic to the real $2^n \times 2^n$ matrices, whose union is weakly dense in $\mathfrak{R}$. Then α is the limit of the transpose maps on the $\mathfrak{R}_n$'s. In order to find this sequence $\mathfrak{R}_n$ we will have to modify the proof of the fundamental theorem of Connes [3, Theorem 5.1], in which he gave several equivalent conditions for a II_1-factor to be hyperfinite. In our proof the factor N in his theorem will be replaced by $\mathfrak{R}$, and the relevant operators will be in $\mathfrak{R}$ rather than N. It will also be necessary to modify the results of McDuff [5] for our purpose, and the classical result of Murray and von Neumann showing the uniqueness of the hyperfinite factor [4, Ch. III, §7, Théorème 3]. Since a complete proof of all this will be too long we shall first prove the necessary lemmas needed, and from then on just indicate the modifications required in order to prove that the algebra $\mathfrak{R}$ defined by α is hyperfinite.

In a recent letter T. Giordano and V. Jones have informed me that they have also shown the uniqueness of the conjugacy classes of involutory *-anti-automorphisms of M, their proof being quite different from mine.

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