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A SMOOTHING PROPERTY OF THE HENKIN AND SZEGÖ PROJECTIONS

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Let D be a bounded strictly psuedoconvex domain in $\mathbf{C}^n$ with smooth boundary; $D = \{z : \rho(z) < 0\}$, where $d\rho(z)$, the gradient of ρ, is not 0 for $z \in \partial D$. The vector field

$$i\bar{\partial}\rho(z) = \left(i\frac{\partial\rho}{\partial\bar{z}_1}(z), \ldots, i\frac{\partial\rho}{\partial\bar{z}_n}(z)\right)$$

is tangent to ∂D for all $z \in \partial D$. Suppose that a smooth tangent vector field r is given on ∂D such that the complex inner product $\langle r(z), i\bar{\partial}\rho(z)\rangle \neq 0$ for any $z \in \partial D$. Following S. Krantz, [6], we say that r is a complex normal vector field. Now suppose that f is a bounded function defined on ∂D and that there is a constant K such that $\|f \circ \gamma\|_\alpha \leqslant K$, for every integral curve γ of the vector field r. Here the norm is the norm in the space of Lipshitz functions of order α. The theorem proved in this paper says that under these conditions the Henkin and Szegö projections of f satisfy a Lipshitz condition of order α in D. In other words, in order that the Henkin or Szegö projection of f satisfy a Lipshitz condition of order α in *every* direction it is sufficient that f itself satisfy a Lipshitz condition of order α only in the direction of the vector field r. The function f need not even be continuous.

This result is a generalization of a theorem proved in [1] and also in [4], which says that if f satisfies a Lipshitz condition of order α on ∂D then the Henkin and Szegö projections of f satisfy a Lipshitz condition of the same order. It can also be regarded as a generalization of a result of W. Rudin [5]. Rudin's result concerns a bounded holomorphic function, f, defined on the unit ball, B, in $\mathbf{C}^n$. For $\zeta \in \partial B$ and $z \in \mathbf{C}, |z| < 1$, he defines $F_\zeta(z) = f(\zeta z)$. He then shows that if F_ζ satisfies a Lipshitz condition of order α, uniformly in ζ, then f satisfies a Lipshitz condition of order α. Rudin's result follows from ours because if f satisfies the hypotheses of Rudin's theorem then its boundary values satisfy the hypotheses of our theorem, with $r(z) = iz$, and f is the Szegö projection of its boundary values. There is a related result due to S. Krantz, [6]. Krantz shows that if f is bounded and holomorphic in a domain D, (not necessarily pseudoconvex), and f satisfies a Lipshitz condition of order α on integral curves of some complex normal vector field r on ∂D, then f satisfies a Lipshitz condition of order α in D. Actually, Krantz does not require that the vector field r exist globally on ∂D.

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