Vol. 47, No. 1 DUKE MATHEMATICAL JOURNAL © March 1980

WHITTAKER MODELS FOR REAL GROUPS

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Introduction. Whittaker functions were first introduced for the principal series representations of Chevalley groups by H. Jacquet [3]. Later, they were pursued by G. Schiffmann for algebraic groups of real rank one [12]. They played a very important role in the development of the Hecke theory for GL_n through the work of H. Jacquet, R. P. Langlands, I. I. Piatetski-Shapiro, and J. A. Shalika [4, 5]. More precisely, they were the main tools for the definitions of local and global L-functions and ϵ-factors. They also appeared quite useful in the development of the Hecke theory for other groups (cf. [10]), as well as in the definition of the local γ-factors of certain functional equations [13, 14], particularly in their factorization. There seems to be other evidence of interest, especially in the work of W. Casselman, B. Kostant [7], and G. Zuckerman.

The analytic behavior of these functions is much simpler when the ground field is non-archimedean; a good account of their analytic properties and some interesting formulas for certain class of such functions may be found in a recent paper of W. Casselman and J. A. Shalika [2]. But when the ground field is archimedean, these functions were believed to behave in a rather complicated manner. In fact, this has been one of the main obstacles in the development of the Hecke theory for number fields.

To make a more precise statement of the problem, we let G be a split reductive algebraic group over $\mathbb{R}$. We fix a maximal torus T of G and we let B be a fixed Borel subgroup of G containing T. We write $B = M_0AU$, the Langlands decomposition of B with $T = M_0A$, and fix a non-degenerate (unitary) character χ of U (see section 1).

Now, let π be a continuous representation of G on a Fréchet space V. Denote by (π_∞, V_∞) the corresponding differentiable representation. Topologize V_∞ with the relative topology inherited from $C^\infty(G, V)$. Let V_K be the subspace of K-finite vectors of V, where K is a fixed maximal compact subgroup of G with $G = KB$. We say that the representation (π, V) is non-degenerate, if there exists a continuous linear functional λ on V_∞, called a Whittaker functional, such that

$$\lambda(\pi(u)v) = \chi(u)\lambda(v) \qquad (u \in U, v \in V_\infty).$$

Then for every $v \in V_\infty$, the Whittaker function W_v is defined to be

$$W_v(g) = \lambda(\pi(g^{-1})v).$$

Received December 20, 1978. Revision received October 15, 1979. Supported by NSF Grant MCS 77-02181.