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SPECTRAL PROPERTIES OF SCHRÖDINGER OPERATORS AND TIME-DECAY OF THE WAVE FUNCTIONS RESULTS IN $L^2(\mathsf{R}^m)$, $m \geqslant 5$

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1. Introduction. We consider Schrödinger operators $H = -\Delta + V$ in $L^2(\mathsf{R}^m)$, $m \geqslant 5$, where V is a (noncentral) potential of moderately short range. Roughly, $V(x) = O(|x|^{-\beta})$ as $x \to \infty$ with $\beta > 2$, but some local singularities will be permitted in the first part of the paper. The present paper is a continuation of [4], where the case $m = 3$ was discussed.

In the first part of the paper we analyze the spectral properties of H in the *low energy limit*. We obtain asymptotic expansions for the resolvent $R(\zeta) = (H - \zeta)^{-1}$ and the spectral density $E'(\lambda)$. Expansions for the scattering matrix $S(\lambda)$ could be obtained, as in [4], but are omitted here. The results are obtained using the integral kernel for the free resolvent $R_0(\zeta) = (-\Delta - \zeta)^{-1}$. Therefore the cases m odd and m even have to be considered separately. The expansions take the following form.

For m odd, $m \geqslant 5$,

$$R(\zeta) = -\zeta^{-1}B_{-2} - i\zeta^{-1/2}B_{-1} + B_0 + \cdots \tag{1.1}$$

$$\pi E'(\lambda) = \operatorname{Im} R(\lambda + i0) = -\lambda^{-1/2}B_{-1} + \lambda^{1/2}B_1 + \cdots \tag{1.2}$$

where $\operatorname{Im}\zeta \geqslant 0$, $\operatorname{Im}\zeta^{1/2} \geqslant 0$, $\lambda = \operatorname{Re}\zeta$ and $\zeta \to 0$.

For m even, $m \geqslant 6$,

$$R(\zeta) = \zeta^{-1}B^0_{-1} + \ln\zeta\, B^1_0 + B^0_0 + \zeta(\ln\zeta)^2 B^2_1 + \zeta\ln\zeta\, B^1_1 + \zeta B^0_1 + \cdots \tag{1.3}$$

$$\pi E'(\lambda) = \operatorname{Im} R(\lambda + i0) = \lambda^{m/2-3}C^0_0 + \lambda^{m/2-2}\ln\lambda\, C^1_1 + \lambda^{m/2-2}C^0_1 + \cdots \tag{1.4}$$

where $\operatorname{Im}\zeta \geqslant 0$, $\lambda = \operatorname{Re}\zeta$ and $\zeta \to 0$.

The expansions are valid in the operator norm in

$$B(-1, s; 1, -s') = B\big(H^{-1,s}(\mathsf{R}^m), H^{1,-s'}(\mathsf{R}^m)\big)$$

where $H^{t,s}(\mathsf{R}^m)$ is the weighted Sobolev space. There is a complicated relation between β, the order of the expansions (1.1)–(1.4), and s, s'. Generally expansions to higher orders require larger β and s, s'.

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