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# NORMAL AUTOMORPHISMS OF ABSOLUTE GALOIS GROUPS OF $\mathfrak{p}$-ADIC FIELDS

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**Introduction.** A (topological) automorphism $\sigma$ of a profinite group $G$ is said to be *normal* if $N^\sigma = N$ for every normal closed subgroup $N$ of $G$; it is said to be *families preserving* if for all $g \in G$, the closed subgroup $\langle g^\sigma \rangle$ generated by $g^\sigma$ is conjugate in $G$ to $\langle g \rangle$. Finally $\sigma$ is called *point-wise inner* if $g^\sigma$ is conjugate to $g$ for every $g \in G$.

The groups of all normal automorphisms, families preserving automorphisms, point-wise inner automorphisms and inner automorphisms of $G$ are denoted by $\mathrm{Aut}_n(G)$, $\mathrm{Aut}_f(G)$, $\mathrm{Aut}_c(G)$ and $\mathrm{Aut}_i(G)$, respectively. Clearly $\mathrm{Aut}_i(G) \leqslant \mathrm{Aut}_c(G) \leqslant \mathrm{Aut}_f(G) \leqslant \mathrm{Aut}_n(G)$.

Neukirch proved in [17] that every automorphism of the absolute Galois group, $G(\mathbf{Q})$, of $\mathbf{Q}$ is normal. Applying Representation Theory of finite groups and a theorem of Scholz [19], Ikeda continued this result and proved in [7] that $\mathrm{Aut}_n(G(\mathbf{Q})) = \mathrm{Aut}_c(G(\mathbf{Q}))$. This was also done, in a different way, by Komatsu in [14]. Then Uchida in [21], Iwasawa in [10], and Ikeda in [8] and [9] have finally proved the famous conjecture of Neukirch, namely that every automorphism of $G(\mathbf{Q})$ is actually inner. Beyond this Ikeda showed in [9] that if $p$ is a prime and $K$ is a finite extension of $\mathbf{Q}_p$, then every point-wise inner automorphism of $G(K)$, the absolute Galois group of $K$, is inner. The main purpose of this note is to strengthen this result and to prove:

THEOREM A. *If $K$ is a finite extension of $\mathbf{Q}_p$, then every normal automorphism of $G(K)$ is inner.*

An analogous result was obtained in [12], where it was concluded from the main result that every normal automorphism of a non-abelian free profinite group is inner. Here we generalize this result in the following way.

We call a class of finite groups *full* if it is closed under the formation of subgroups, homomorphic images and group extensions. Examples of full classes are the class of all finite groups, all $p$-groups and all solvable finite groups. Let $\mathcal{C}$ be a full class of finite groups. A *pro-$\mathcal{C}$-group presented by e generators and d word-relations* is the pro-$\mathcal{C}$-completion of a discrete group presented by $e$ generators and $d$ relations in these generators. We prove:

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