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INVARIANTS OF THE NPT ISOTOPY CLASSES OF MORSE-SMALE DIFFEOMORPHISMS OF SURFACES

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We are concerned with the evolution of structurally stable systems. Under what circumstances can one structurally stable system evolve to another while undergoing only a finite number of easily describable changes (bifurcations)? For example, if the systems are Morse-Smale flows, then Newhouse and Peixoto [6] have shown that such an evolution is always possible. (See Newhouse [4] for an extension of this result.)

If the systems are generated by a diffeomorphism, very few general results are known. Two orientation preserving Morse-Smale diffeomorphisms of S^1 can be connected by an arc with only finitely many bifucations if and only if they have the same rotation number. In fact, if they have the same rotation number, they can be connected by a *stable* arc (one whose qualitative features do not change under any sufficiently small perturbation).

We are concerned here with diffeomorphisms of closed surfaces. In particular, we are interested in determining when two Morse-Smale diffeomorphisms of a surface can be connected by an NPT arc. These arcs, originally studied by Newhouse, Palis, and Takens [5], can be modified to be stable and have only a finite number of bifurcations. Furthermore, it is conjectured that any stable arc is an NPT arc [5]. Given a Morse-Smale diffeomorphism we construct a "periodic decomposition" of the manifold. Our principal result states that the respective decompositions resulting from two NPT isotopic Morse-Smale diffeomorphisms are related in a simple way. As a consequence, we deduce the existence of infinitely many NPT isotopy classes within each isotopy class containing an orientation preserving Morse-Smale diffeomorphism.

The first section of this paper sketches some background material and presents the definition of an NPT arc. The main results are stated in the second section. In that section, given any orientation preserving Morse-Smale diffeomorphism g, we construct infinitely many Morse-Smale diffeomorphisms isotopic to g with the property that no two of these diffeomorphisms are NPT-isotopic. Section Three contains the construction of the periodic decomposition as well as the proof of a uniqueness result. In Section Four we show how the periodic decomposition is related to an NPT arc.

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