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GALOIS GROUPS OF ENUMERATIVE PROBLEMS

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0. Introduction. This paper is concerned with the solvability of certain enumerative problems in algebraic geometry. To illustrate the questions raised, consider one of the problems dealt with below—that of locating the flexes of a plane curve. We let $C \subset \mathbf{P}^2$ be a complex plane curve of degree d, given as the locus of the homogeneous polynomial $F(X_0, X_1, X_2) = \sum a_{ij} X_0^{d-i-j} x_1^i x_2^j$ (or in euclidean coordinates $x_i = X_i / X_0$, as the locus of $f(x_1, x_2) = \sum a_{ij} x_1^i x_2^j$); we will take coefficients a_{ij} to be general complex numbers. At a generic point of C, then, the tangent line $l = T_p(C)$ intersects C with multiplicity $m_p(C \cdot l) = 2$; we say that p is a *flex point* of C if $m_p(l \cdot C) \geqslant 3$. An elementary count of parameters leads us to expect that C will have a finite number of flex points p_α; and accordingly we may ask two questions: first, how many flexes does C possess? and second, can we find them?—that is, is it possible to give a formula for coordinates $x_i(p_\alpha)$ of the flex points p_α of C in terms of the coefficients a_{ij}?

There are a number of ways of answering the first of these questions; historically the first was to note that the flexes of C comprise the locus $F(X) = H(X) = 0$, where $H(X)$ is the *Hessian*

$$H(X) = \det\left(\frac{\partial^2 F}{\partial X_i \partial X_j} \right);$$

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