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GENERIC UNFOLDINGS AND NORMAL FORMS OF SOME SINGULARITIES ARISING IN THE OBSTACLE PROBLEM

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§1. **Introduction**. This paper is a continuation of a previous one [11] in which a bifurcation analysis was made for certain singularities in the free boundary arising in an obstacle problem. Our work is motivated by results of Schaeffer [12], [13], who studied how free boundaries vary under perturbations of the given data, and who pointed out the need for a generic theory of such variations. Results of Kinderlehrer and Nirenberg [9] show such a theory would be of a very special nature, as there are rather severe restrictions on the singularities that can actually occur.

Our point of view is to develop a framework wherein such problems can be analyzed using the techniques of bifurcation theory. In [4] this was done for a particular singularity. The emphasis in [11] and in the present paper is toward a local study of such singularities and their normal forms, along with generic hypotheses and bifurcation diagrams. We do not study the global obstacle problem itself, but rather the class of local singularities arising from it. In effect, we are presupposing enough regularity of the free boundary, and in how it deforms, that the resulting singularities can be described by a suitably regular class of functions.

The philosophy and justification for such an approach was described in some detail in [11]; it is based in part upon regularity results for free boundaries, such as those of [1], [8], and [10]. The question of relating information about the singularities back to the obstacle problem was also discussed in [11]; this proceeds in the spirit of "restricted unfoldings", as in [2], [3], [5], [6], and [7]. We refer the reader to [11] for a more detailed discussion of our philosophy and motivation as well as for the more technical aspects of the problem.

Let us summarize the setting developed in [11]. Briefly, we are concerned with local deformations of a singular curve

$$y^2 = p_0(x), \qquad (x, y) \in \mathbb{R}^2 \quad \text{near } (0, 0) \tag{1.1}$$

in the plane; $p_0(x)$ is real analytic with a zero of order n at the origin; we write

$$p_0(x) = x^n b_0(x)^2, \qquad b_0(0) > 0,$$

$$b_0 \text{ is real analytic near } x = 0.$$

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