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DECOMPOSITIONS OF H_p SPACES

P. WOJTASZCZYK

In the present paper we study the linear-topological structure of H_p spaces, $1 \leqslant p \leqslant \infty$ and the disc algebra A. Our main result is the following theorem.

THEOREM 1. *The space H_p, $1 \leqslant p \leqslant \infty$ is isomorphic to $(\sum_{n=1}^{\infty} H_p)_p$ and the disc algebra A is isomorphic to $(\sum_{n=1}^{\infty} A)_{c_0}$.*

This theorem is new for the space H_1 and A only. The space H_∞ was considered in [9], but we feel that the present proof is simpler. For $1 < p < \infty$, the theorem is well known but our proof gives some new information (cf. Remark 2). As an application we consider the problem of B. S. Mitiagin ([7] Conjectures 1, 2, 3) about the homotopy properties of the general linear group $GL(A)$ of the disc algebra A. We are able to show that $GL(A)$ is contractible (Theorem 2). Another application of our technique is that the space L_1/H_1 is isomorphic to $(\sum_{n=1}^{\infty} L_1/H_1)_1$ (Theorem 4). This enables us to answer some problems of Pełczyński [8].

Our methods are influenced by methods of [9] and by some unpublished work of F. Delbaen (cf. Remark 5). Our methods readily generalise to H_p spaces on polydiscs in an n-dimensional complex space. Since we have described such generalisations in some detail in [9], the present paper is devoted to functions of one complex variable. Only in remarks do we state more general theorems.

Let us now establish some notational conventions used in this paper. $\mathbf{C}$ denotes the complex plane. We will constantly use

$$\mathbf{T} = \{z \in \mathbf{C} : |z| = 1\}$$

and

$$\mathbf{D} = \{z \in \mathbf{C} : |z| < 1\}.$$

For an interval $I \subset \mathbf{T}$ its length will be denoted by $|I|$.

The spaces we consider are spaces of analytic functions on $\mathbf{D}$. H_p, $1 \leqslant p \leqslant \infty$, is the space of all functions such that

$$\|f\|_{H_p} = \sup_{0<r<1} \left(1/2\pi \int_0^{2\pi} |f(re^{i\theta})|^p \, d\theta\right)^{1/p} < \infty.$$

The disc algebra A is the space of all functions continuous on $\bar{\mathbf{D}}$ and analytic in $\mathbf{D}$. It is equipped with the supremum norm.

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