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ON THE CLASSIFICATION OF THE ORDERED GROUPS ASSOCIATED WITH THE APPROXIMATELY FINITE DIMENSIONAL C^*-ALGEBRAS

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Introduction. The algebraic classification of C^*-algebras has proved to be a difficult problem. To date, there are only two broad classes of C^*-algebras for which we have detailed structural information: The GCR (or "type I") algebras of Kaplansky, and the approximately finite dimensional (or "AF") algebras considered by Glimm [9], Dixmier [2], Bratteli [1] and Elliott [6]. In the latter article it was proved that the (stable) classification of the AF algebras can be reduced via K-theory to the classification of certain discrete, ordered abelian groups (see also Effros and Rosenberg [4]). In particular, Elliott showed that the ordered groups that arise are just those that can be written as inductive limits of systems of groups of the form $\mathbf{Z}^n$, where the latter is ordered by the cone

$$(\mathbf{Z}^n)^+ = \{(a_1, \ldots, a_n) : a_i \in \mathbf{Z}^+\}.$$

Following Elliott, we shall call such limits *dimension groups*. (Strictly speaking, for the applications to AF algebras, we should only consider countable groups and totally ordered sequences of $\mathbf{Z}^n$'s, but this situation is easily handled by the general method.)

The dimension groups satisfy two simple properties, which in fact might also characterize these groups (this was shown by Elliott in certain special cases [7]. The relevance of the second property was independently pointed out to the author by Professor Effros). To be specific, given an ordered abelian group G (see §1), we say that

(1) G is *isolated ordered* if for $g \in G$, $n \in \mathbf{N}$, $ng \geqslant 0$ implies that $g \geqslant 0$,

(2) G satisfies the *Riesz interpolation property* if for $u, v \leqslant x, y$, there exists w such that $u, v \leqslant w \leqslant x, y$.

Ordered groups satisfying these properties were first investigated by Fuchs [8]. Slightly modifying his definition, we shall say that an ordered group is a *Riesz group* if it satisfies both of these properties (this differs from the terminology of Fuchs and Elliott who do not include the first condition). The interpolation property should be considered as a weakening of the concept of lattice ordering (see §1), since we could let $w = u \vee v$ if the latter existed. The importance of this broader notion is that very simple constructions lead one to Riesz groups that are not lattice ordered (see the remark at the end of §1).

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