Vol. 46, No. 3 DUKE MATHEMATICAL JOURNAL © September 1979

SPECTRAL PROPERTIES OF SCHRÖDINGER OPERATORS AND TIME-DECAY OF THE WAVE FUNCTIONS

ARNE JENSEN AND TOSIO KATO

1. Introduction. This paper is concerned with the Schrödinger operator $H = -\Delta + V(x)$ in $\mathcal{H} = L^2(R^3)$, where $V(x)$ is assumed to be a (noncentral) potential of moderately short range. Roughly speaking, we assume that $V(x) = 0(|x|^{-\beta})$ for large $|x|$ with a finite $\beta > 0$, but some local singularities will be permitted in the first part of the paper. For example,

$$(1 + |x|)^{\beta} V(x) \in L^{3/2}_{ul}(R^3), \qquad \beta > 2, \tag{1.1}$$

is a convenient condition; $f \in L^p_{ul}$ (uniformly local L^p-space) means that the L^p-norm of f on a ball of unit radius is bounded independent of the position of the ball. ($H = -\Delta + V$ is defined as the form sum.)

In the first part of the paper, we analyze the spectral properties of H in the *low energy limit*. More specifically, we deduce asymptotic expansions for the resolvent $R(\zeta) = (H - \zeta)^{-1}$, the spectral density $E'(\lambda)$ and the S-matrix $S(\lambda)$. These expansions take the form

$$R(\zeta) = -\zeta^{-1}B_{-2} - i\zeta^{-1/2}B_{-1} + B_0 + i\zeta^{1/2}B_1 + \cdots, \tag{1.2}$$

$$\pi E'(\lambda) = \operatorname{Im} R(\zeta) = -\lambda^{-1/2}B_{-1} + \lambda^{1/2}B_1 + \cdots, \tag{1.3}$$

$$S(\lambda) = \Sigma_0 + i\lambda^{1/2}\Sigma_1 - \lambda\Sigma_2 \cdots, \tag{1.4}$$

where $\operatorname{Im} \zeta \geqslant 0$, $\operatorname{Im} \zeta^{1/2} \geqslant 0$, $\lambda = \operatorname{Re} \zeta > 0$ and $|\zeta| \to 0$.

For (1.2) and (1.3), it is necessary to use a topology different from the usual one for operators in $\mathcal{H}$. As shown by the work of Agmon [1] and Kuroda [6] a convenient choice is the operator norm in

$$\mathfrak{B}(m, s; m', s') = \mathfrak{B}\big(H^{m,s}(R^3), H^{m',s'}(R^3)\big), \tag{1.5}$$

where $H^{m,s}(R^3)$ denotes the *weighted Sobolev space*, with the associated norm

$$\|u\|_{H^{m,s}} = \|(1 + |x|^2)^{s/2}(1 - \Delta)^{m/2}u\|_{L^2}. \tag{1.6}$$

Received March 19, 1979. Revision received May 4, 1979. Second author partially supported by NSF Grant MCS76-04655.