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MICROLOCAL STRUCTURE OF INVOLUTIVE CONICAL REFRACTION

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1. Introduction. The phenomenon of conical refraction (see Courant & Hilbert [3]), which is the splitting of a ray, by a biaxial crystal, into a cone of rays is attributable to the non-uniform multiplicity of Maxwell's equations in the crystal. Here, we investigate single operators which exhibit similar behavior. Explicitly, we suppose P to be a classical pseudodifferential operator of order m on a C^∞ manifold M and $\rho \in T^*M\backslash 0$ to be a base point near which the following conditions hold:

(1.1) P has real principal symbol p.

Let $\Sigma = \{\rho' \in T^*M\backslash 0;\ p(\rho') = 0\}$ and $\Sigma_2 = \{\rho' \in \Sigma;\ dp(\rho') = 0\}$.

(1.2) Σ_2 is a non-radical involutive submanifold of codimension $k \geqslant 3$ through ρ.

The Hessian bilinear form of p, $\mathrm{Hess}(p)(\rho') : T_{\rho'}(T^*M) \times T_{\rho'}(T^*M) \to \mathbb{R}$ is invariantly defined for every $\rho' \in \Sigma_2$. To fix the form of Σ near Σ_2 we require

(1.3) $\mathrm{Hess}(p)(\rho)$ has rank k and positivity 1.

In view of (1.2) this condition persists for $\rho' \in \Sigma_2$ near ρ. A necessary and sufficient condition for the well-posedness of the Cauchy problem for a second order hyperbolic operator satisfying global versions of (1.2), (1.3), the Levi condition, was obtained by Ivrii & Petkov [6], Hörmander [4] and we assume a microlocal version of it:

$$\sigma_{\mathrm{sub}}(P)|_{\Sigma_2} = 0 \quad \text{near } \rho. \tag{1.4}$$

The subprincipal symbol is well defined as a function on T^*M if P acts on half densities, otherwise $\sigma_{\mathrm{sub}}(P)$ will be a section of a line bundle, so (1.4) is meaningful.

Under assumptions (1.1)–(1.4) we construct two distinguished microlocal parametrices for P at ρ. The submanifold Σ_2, being involutive, has a natural k-foliation on the leaves of which the Hessian of p induces a Lorentzian structure (i.e., a pseudo-Riemannian structure of positivity 1). Once we select a local forward "time" direction on the leaf through ρ, and therefore on all leaves near ρ, we can define $C_2^+ \subset \Sigma_2 \times \Sigma_2$, the forward double bicharacteristic relation, by admitting $(\rho'', \rho') \in C_2^+$ when ρ'' is on the same leaf as ρ' and

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