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RING SPECTRA WHICH ARE THOM COMPLEXES

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For our purposes a ring spectrum E is a spectrum with a map $i : E \wedge E \to E$ and a unit $i : S^0 \to E$ such that the following diagrams commute up to homotopy:

$$\begin{array}{ccc} E \wedge E \wedge E & \xrightarrow{\mu \wedge 1} & E \wedge E \\ \downarrow {\scriptstyle 1 \wedge \mu} & & \downarrow {\scriptstyle \mu} \\ E \wedge E & \xrightarrow{\mu} & E \end{array} \qquad \begin{array}{ccccc} S^0 \wedge E & \xrightarrow{i \wedge 1} & E \wedge E & \xleftarrow{1 \wedge i} & E \wedge S^0 \\ & {\scriptstyle l} \searrow & \downarrow {\scriptstyle \mu} & \swarrow {\scriptstyle r} & \\ & & E & & \end{array}$$

The ring spectrum is abelian if

$$\begin{array}{ccc} E \wedge E & \xrightarrow{T} & E \wedge E \\ & {\scriptstyle \mu} \searrow \quad \swarrow {\scriptstyle \mu} & \\ & E & \end{array}$$

commutes up to homotopy where T is the map that exchanges factors.

Let L be a space and let ξ be a fibration over L classified by a map $f : L \to BF$ (the classifying space of stable spherical fibrations). We can form the Thom spectrum $T(f)$ of f as a suspension spectrum by letting $(T(f))_n$ be the Thom complex of $L^n \to BF_n$ where L^n is the n-skeleton of L. This makes $T(f) = \{(T(f))_n\}$ into a suspension spectrum.

Spectra which arise in this fashion have a unit which is the inclusion of the fiber on the Thom class.

Natural examples of maps $f : L \to BF$ give a plethora of interesting spectra: among them are $K(\mathbf{Z}_2, 0)$, $K(\mathbf{Z}, 0)$, the Brown-Gitler spectrum, and a spectrum for which the secondary operation of Adams $\varphi_{j,j}$ [1] is defined and non-zero on the Thom class.

Frequently, the Thom spectra which we obtain in this manner are commutative ring spectra. A useful feature of these Thom spectra is that they admit particularly nice resolutions. Consequently, these spectra give rise to

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