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A NOTE ON BOCHNER-RIESZ OPERATORS

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The purpose of this note is to continue the program initiated in [2] and here we prove how the theorem for the Kakeya maximal function implies the two-dimensional result for the spherical summation operators obtaining, therefore, an alternative approach to the result of L. Carleson and P. Sjölin [1] and C. Fefferman [4]. The proof will follow the main lines of the paradigm of the Calderón-Zygmund's theory: we start with the Fourier multiplier defined in R^2 by the formula

$$\widehat{T_\lambda f}(\xi) = (1 - |\xi|^2)_+^\lambda \cdot \hat{f}(\xi), \qquad \lambda > 0;$$

the geometry of the situation suggests a decomposition of T_λ into pieces

$$T_\lambda = \sum_\alpha S_\alpha;$$

then we introduce two adequate g-functions to reduce the study of T_λ to that of

$$Mf(x) = \operatorname*{Sup}_\alpha |S_\alpha f(x)|$$

and, finally, we reduce the analysis of the maximal operators M to a problem in Euclidean geometry, by considering its dual operators M^* and interpreting it as a covering lemma for an appropriate family of rectangles.

Results. (a) Suppose that $\phi : \mathsf{R} \to \mathsf{R}$ is a smooth function supported in $[-1, +1]$ and consider the family of Fourier multipliers S_δ, where $\delta > 0$ is small, defined by the formula

$$\widehat{S_\delta f}(\xi) = \phi(\delta^{-1}(|\xi| - 1)) \cdot \hat{f}(\xi),$$

for rapidly decreasing smooth functions f.

THEOREM 1. *There exists a constant C, independent of δ, such that*

$$\| S_\delta f \|_4 \leqslant C(\log 1/\delta)^{1/4} \| f \|_4$$

for every $f \in S(\mathsf{R}^2)$.

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