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PURE POINT SPECTRUM AND NEGATIVE CURVATURE FOR NONCOMPACT MANIFOLDS

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1. Introduction. Let $q(x)$ be a real valued continuous function on the real line R. Suppose that $\lim q(x) = \infty$ as $|x| \to \infty$. In his famous paper [11], H. Weyl showed that, for q monotonic in $|x|$, the unbounded operator $-d^2/dx^2 + q(x)$, acting on $L^2(R)$, has pure point spectrum. Titchmarsh [9] showed that it is possible to remove the monotonicity hypothesis on q.

The present paper is devoted to some analogous theorems for Riemannian manifolds. Suppose that M is a complete Riemannian manifold. The Laplacian of M will be denoted by Δ. Fix a point $p \in M$ and write $K(r) = \sup\{K(x, \pi) \mid d(p, x) \geqslant r\}$ where π is a two plane in T_xM. Then one has

THEOREM I.1. *Let M be a complete simply connected negatively curved Riemannian manifold. If* $\lim K(r) = -\infty$ *as* $r \to \infty$, *then* Δ *has pure point spectrum.*

For the special case of surfaces we can ease the topological restrictions to obtain:

THEOREM I.2. *Let M be a complete surface with finitely generated fundamental group $\pi_1(M)$. If* $\lim K(r) = -\infty$ *as* $r \to \infty$, *then* Δ *has pure point spectrum.*

The main concern of the present paper is to establish Theorems I.1, I.2. In a final section, we explain more clearly the relationship to the Titchmarsh-Weyl theorem.

2. The decomposition principle. Let M be a complete Riemannian manifold without boundary. The Laplacian Δ of M is formally self adjoint on $C_0^\infty(M)$. Since M is complete, it is well known [4] that Δ has a unique self-adjoint extension, also denoted Δ, to an unbounded operator on $L^2(M)$. Moreover, the domain of Δ consists of those $f \in L^2(M)$ such that $\Delta f \in L^2(M)$ in the sense of distributions. If $N \subset M$ is a compact manifold with boundary, of the same dimension as M, then one obtains a self-adjoint extension Δ' of the Laplacian Δ of $M - N$ by imposing Dirichlet boundary conditions.

By analogy with [5], one has the following:

PROPOSITION 2.1. (Decomposition Principle). *Let M, N be as above. Then the Laplacians Δ, Δ' have the same continuous spectrum.*

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