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ESTIMATE ON THE FUNDAMENTAL FREQUENCY OF A DRUM

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Recently, there has been some interest in estimating from below the smallest eigenvalue Λ_1^2 of $-\Delta$ on a connected domain $\Omega \subset \mathbb{R}^2$ with Dirichlet boundary conditions. In particular, the following result has been obtained by Hayman [2].

THEOREM 1. *If Ω is simply connected, then*

$$\Lambda_1 \geqslant \frac{c_0}{\rho}.$$

Here ρ is the supremum of the radius of a disc that can be placed in Ω. c_0 is an absolute constant; Hayman has $c_0 = 1/30$. In this note we will establish the following generalization.

THEOREM 2. *If Ω is k-connected, then*

$$\Lambda_1 \geqslant \frac{c_1}{\sqrt{k}\,\rho}$$

where c_1 is an absolute constant.

The proof will be based on the following eigenvalue estimate, which we established in [5].

THEOREM A. *Let Q be the unit square in $\mathbb{R}^2$ (or the unit cube in $\mathbb{R}^n$), $K \subset \overline{Q}$ a compact subset. Then the smallest eigenvalue μ_1 of $-\Delta$ on $Q \setminus K$, with Dirichlet boundary conditions on ∂K and Neumann boundary conditions on $\partial Q \setminus \partial K$, satisfies the estimate*

$$\mu_1 \geqslant c_2 \operatorname{cap} K$$

for some absolute constant c_2 (depending only on n).

Osserman in [4] has proved a version of theorem 2, namely $\Lambda_1 \geqslant c_1'/k\rho$. For large k, our theorem is an improvement of his result. Due to inertia on the part of the author, the absolute constant c_1 has not been evaluated, but probably for k small, Osserman's result is sharper than ours. Osserman has $c_0 = \frac{1}{2}$, $c_1 = 1$ $(k \geqslant 2)$.

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