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FLAT SOLUTIONS AND SINGULAR SOLUTIONS OF HOMOGENEOUS LINEAR PARTIAL DIFFERENTIAL EQUATIONS WITH ANALYTIC COEFFICIENTS

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§0. Introduction. We deal with a linear partial differential operator $P = P(x, D)$ of order $m \geqslant 1$, with (complex) coefficients defined and analytic in an open subset Ω of $\mathbb{R}^n$. The question that interests us in the present article is whether one can find solutions of the homogeneous equation

$$P(x, D)u = 0 \tag{0.1}$$

with zeros of infinite order or, alternatively, with singularities, concentrated on an analytic submanifold M of Ω. (We say that a C^∞ function is *flat* on M if all its derivatives vanish there.)

This is of course a topic on which many results are known, beginning with the classical Holmgren's theorem—which concerns noncharacteristic hypersurfaces. Let us introduce some terminology and notation: By $T^*\Omega\backslash 0$ we denote the cotangent bundle over Ω from which the zero section has been excised; by $N^*(M)$ we mean the conormal bundle over M, which is the subset of $T^*\Omega\backslash 0$ consisting of the points (x, ξ) with x in M and ξ orthogonal to all tangent vectors to M at x; by Char P the *characteristic set* of P, i.e., the subset of $T^*\Omega\backslash 0$ where the principal symbol of P, $p_m(x, \xi)$, vanishes. We shall say that M is *noncharacteristic* (with respect to P, but this may be omitted as no other differential operator will be considered) if $N^*(M)$ and Char P do not intersect,

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