

## IRREDUCIBLE CHARACTERS OF SEMISIMPLE LIE GROUPS I

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**1. Introduction.** Let  $G$  be a connected semisimple Lie group. In [16], a collection of problems in the representation theory of  $G$  was set forth; one of the major ones was the determination of the irreducible characters of  $G$ . (This problem is not solved in the present paper.) The main theorem of [16] (Theorem 3.7 and Lemma 3.11 below) is a simple description of how these characters behave under “coherent continuation” (defined by 2.6 below). From this theorem, reducibility conditions for some standard induced representations were obtained. Unfortunately, the proof given in [16] for Theorem 3.7 is extremely complicated, and not very enlightening. The first purpose of this paper is to present a greatly simplified proof of this theorem, using Duflo’s realization of the primitive ideals in the enveloping algebra of the complexified Lie algebra  $\mathfrak{g}$  of  $G$  (cf. [4]).

Our study of irreducible characters is along the lines sketched in [16]. As indicated there (in Theorem 6.18) it suffices to determine the irreducible characters with a given nonsingular infinitesimal character. These form a finite set  $A$ , which has been parametrized by Langlands in [11]. Thus we may write  $A = \{\Theta_1, \dots, \Theta_N\}$ ; here each  $\Theta_i$  is characterized in a certain way, but not explicitly known. To each  $\Theta \in A$ , a finite set  $\{U_\alpha(\Theta)\}$  of invariant eigendistributions was associated in [16]; this will be described in section 3 (Definition 3.8). Each  $U_\alpha(\Theta)$  is a sum of elements of  $A$ , with non-negative integral multiplicities. Once these multiplicities are known (for all  $\alpha$  and  $\Theta$ ) the  $\Theta_i$  can be computed explicitly by a finite algorithm. (This algorithm will be given in Section 5.) So the problem we consider is the determination of these multiplicities. (For reasons discussed in Section 2, this seems to be the simplest way of describing the irreducible characters.) Some of the multiplicities were found in [16]. Here we determine some more of them (Theorems 4.12 and 4.14). More importantly, we give a result relating the multiplicities to the dimensions of certain Ext groups (Theorem 3.9). This turns out to be an extremely powerful computational tool. In Section 6 we illustrate these results with a computation in  $SP(3, 1)$ .

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