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APPENDIX TO O. BRATTELI'S PAPER ON "CROSSED PRODUCTS OF UHF ALGEBRAS"

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These remarks are an appendix to [1], and were suggested by a problem posed in a preliminary version of that paper. They help to "explain" Bratteli's Proposition 3.4 and the example in his §4. The author would like to acknowledge several helpful conversations with Elliot Gootman concerning conditions for simplicity of crossed products.

PROPOSITION. *Let α be a strongly continuous action of a compact group G as $*$-automorphisms of a C^*-algebra $\mathfrak{A}$. Then the fixed-point algebra $\mathfrak{A}^\alpha$ is canonically isomorphic to a hereditary subalgebra* (*in fact, to a "corner"—see* [3]) *of the crossed product* $C^*(\mathfrak{A}, G)$.

Proof. Let $p : G \to \mathfrak{A}^\sim$ be the constant function whose value everywhere is 1 (here $\mathfrak{A}^\sim$ denotes $\mathfrak{A}$ if $\mathfrak{A}$ has a unit and the algebra obtained from $\mathfrak{A}$ by adjoining an identity otherwise). Then $p \in L^1(\mathfrak{A}^\sim, G)$ (of which $C^*(\mathfrak{A}^\sim, G) \supseteq C^*(\mathfrak{A}, G)$ is the completion), and for $x \in L^1(\mathfrak{A}, G)$ we have

$$(x * p)(t) = \int x(s)\alpha_s\big(p(s^{-1}t)\big)ds = \int x(s)ds = x_0;$$

$$(p * x * p)(r) = \int p(s)\alpha_s\big((x * p)(s^{-1}r)\big)ds = \int \alpha_s(x_0)ds = x_1$$

(for suitable x_0, x_1), so that $p * x * p$ is a constant function whose value everywhere is an element x_1 of the fixed-point algebra $\mathfrak{A}^\alpha$. Now one quickly observes that such constant functions taking values in $\mathfrak{A}^\alpha$ constitute a closed $*$-subalgebra of $L^1(\mathfrak{A}, G)$, isomorphic to $\mathfrak{A}^\alpha$, for which p acts as a unit element. Passing to the C^*-completions, we see that p is a projection in the multiplier algebra of $C^*(\mathfrak{A}, G)$ and that $\mathfrak{A}^\alpha \cong pC^*(\mathfrak{A}, G)p$.

COROLLARY. *Under the conditions of the proposition,* $\operatorname{Prim}(\mathfrak{A}^\alpha)$ *is homeomorphic with an open subset of* $\operatorname{Prim}(C^*(\mathfrak{A}, G))$. *In particular* $C^*(\mathfrak{A}, G)$ *cannot be simple unless* $\mathfrak{A}^\alpha$ *is simple.*

Proof. If $\mathfrak{B} = C^*(\mathfrak{A}, G)$ and p is as above, we have $\mathfrak{A}^\alpha \cong p\mathfrak{B}p$, which is then strongly Morita equivalent to the ideal $\mathfrak{B}p\mathfrak{B}$ of $\mathfrak{B}$ [4, Examples 6.7–6.8]. Thus $\operatorname{Prim}(\mathfrak{A}^\alpha) \cong \operatorname{Prim}(\mathfrak{B}p\mathfrak{B})$ (by [5, Corollary 3.8]), which is an open subset of $\operatorname{Prim}(\mathfrak{B})$.

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