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A TRACE FORMULA FOR REDUCTIVE GROUPS I TERMS ASSOCIATED TO CLASSES IN $G(\mathbf{Q})$

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Introduction

This paper, along with a subsequent one, is an attempt to generalize the Selberg trace formula to an arbitrary reductive group G defined over the rational numbers. Our main results have been announced in the lectures [1(*c*)].

In his original papers [8(*a*), (*b*)], Selberg gave a novel formula for the trace of a certain operator associated with a compact quotient of a semisimple Lie group and a discrete subgroup. When the discrete subgroup is arithmetic, the situation is essentially equivalent to the case that the group G is anisotropic. Then $G(\mathbf{A})$ is a locally compact topological group and $G(\mathbf{Q})$ is a discrete subgroup such that $G(\mathbf{Q})\backslash G(\mathbf{A})$, the space of cosets (with the quotient topology), is compact. The operator is convolution on $L^2(G(\mathbf{Q})\backslash G(\mathbf{A}))$ by a smooth, compactly supported function f on $G(\mathbf{A})$.

Let us recall how to derive the formula in this case. To understand the idea, it is not necessary to be an expert in algebraic groups, or even to be familiar with the notion of adèles. If $\phi \in L^2(G(\mathbf{Q})\backslash G(\mathbf{A})$, define

$$(R(y)\phi)(x) = \phi(xy), \qquad x, y \in G(\mathbf{A}).$$

Then R is a unitary representation of $G(\mathbf{A})$, and the convolution operator is defined by,

$$R(f) = \int_{G(\mathbf{A})} f(y)R(y)dy.$$

$(R(f)\phi)(x)$ can be written

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