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SCHRÖDINGER OPERATORS WITH MAGNETIC FIELDS. I. GENERAL INTERACTIONS

J. AVRON, I. HERBST AND B. SIMON

§1. Introduction

The rigorous study of Schrödinger operators, $-\Delta + V$, has become a highly developed mathematical discipline during the past 25 years, see e.g., [39, 40, 41] for a systematic review. There has been much less study of Schrödinger operators with magnetic fields, i.e., $-(\vec{\nabla} - i\vec{a})^2 + V$, where $\vec{a}$ is the magnetic vector potential so that the magnetic field $\vec{B}$ is given by $\vec{B} = \vec{\nabla} \times \vec{a}$ (in three dimensions; in general dimension, $\vec{B}$ should be thought of as a 2-form). The bulk of the literature allowing magnetic fields requires $\vec{a}$ to go to zero at infinity which is quite far from the important case of constant $\vec{B}$ ($\vec{a}$ is linear). One big exception to this is the work on essential self-adjointness which, since the paper of Ikebe-Kato [25], has placed no restriction on the behavior of $\vec{a}$ at infinity (e.g., Ikebe-Kato allow an arbitrary $\vec{a}$ which is once continuously differentiable); see [44, 50, 45, 66] for recent developments. The only previous studies of "spectral theory" in cases where $\vec{B}$ does not go to zero at infinity are those of Jörgens [27] and Schechter [44] (see also [28, 18, 56] and paper II of our series [6]) locating the essential spectrum in N-body problems when one mass is infinite and the potentials between particles of finite mass are positive, Grossman [21] discussing direct integral decompositions for periodic potentials in magnetic fields and Lavine-O'Carroll [34] who discuss a very special problem (to which we return in §5 and another paper [7]). Simultaneously to our work, Combes, Schrader, and Seiler [11] have studied the semi-classical limit with results that overlap ours on one point (see §2).

This is the first of a series of papers on the spectral and scattering theory of Schrödinger operators with magnetic fields. In this paper we study the theory for general $\vec{a}$'s and V's and also the special case where $\vec{a}$ is linear ($\vec{B}$ constant). In II of the series [6], we describe some rather novel features of the reduction of the center of mass in constant $\vec{B}$ field. In III of the series [7], we describe some special problems associated with the case $\vec{B}$ = constant, V = sum of Coulomb

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