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NONSPANNING SETS OF POWERS ON CURVES: ANALYTICITY THEOREM

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1. Introduction and results.

Let $\{p_n\}$ be an increasing sequence of positive integers. If $\sum 1/p_n$ diverges, every continuous function on $[0, 1]$ can be uniformly approximated by linear combinations of the powers 1 and

$$x^{p_1}, x^{p_2}, \cdots, \tag{1.1}$$

but if $\sum 1/p_n$ converges, these powers do not span $C[0, 1]$ (Müntz [11]; he also considered general positive p_n; complex exponents were considered by Szász [15]). Clarkson and Erdös [2] and Schwartz [13] proved the corresponding result for the powers (1.1) and the space $C[a, b]$ where $a > 0$. More important, they showed that for convergent $\sum 1/p_n$, every uniform limit f on $[a, b]$ of combinations of powers (1.1) has an analytic extension F to the disc $\{|z| < b\}$; the power series for F contains only powers z^{p_n}. For a complete characterization of the approximable functions, see Schwartz [13] and Korevaar [3]. Extensions of these results to more general exponents have been treated also, notably by Schwartz [13]; a method to deal with the case of $C[a, b]$ without going via $C[0, b]$ has been given by Luxemburg and Korevaar [8].

If γ is an arbitrary Jordan arc in the plane, the nonnegative integral powers z^n form a spanning set for $C(\gamma)$ (Walsh [16]). However, there is still no neat Müntz theorem for arcs. It is known that for *analytic* γ, the family of powers

$$z^{p_1}, z^{p_2}, \cdots \tag{1.2}$$

never spans $C(\gamma)$ when $\sum 1/p_n$ converges (Malliavin and Siddiqi [9], Korevaar [4]). For *arbitrary* arcs γ, (1.2) is known to be nonspanning only under stronger conditions, for example,

$$p_n > cn \log n(\log \log n)^{2+\delta}, \qquad (c, \delta > 0) \tag{1.3}$$

or

$$\sum 1/p_n < \infty, \; p_n/n \uparrow \tag{1.3'}$$

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