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A NOTE ON THE BERGMAN KERNEL

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Let U be a bounded domain in $\mathbb{C}^n$ and let $A(U)$ denote the holomorphic functions in $L^2(U)$. Suppose $\{\varphi_a\}$ are an orthonormal basis for $A(U)$. The Bergman kernel function $K(z, w)$ is defined by

1. $$K(z, w) = \Sigma\varphi_a(z)\overline{\varphi_a(w)}$$

K is independent of the choice of φ_a, and satisfies the following well known properties.

2. $K(z, w)$ is holomorphic in z, anti-holomorphic in w.

3. $$\text{If } f \in A(U), \qquad f(z) = \int K(z, w)f(w)dV(w).$$

Suppose also that U has smooth boundary. Let r denote a defining function for U. r is then a C^∞ function satisfying

4. $$r < 0 \text{ on } U,\ r > 0 \text{ outside closure } U,\ r = 0 \text{ on } bd.\ U \text{ and } d\,r \neq 0 \text{ on } bd.\ U.$$

Let

$$T^{10}(bd\ U) = \left\{\text{Vector fields } L = \sum a_i \frac{\partial}{\partial z_i} \quad \text{with} \quad \langle dr, L\rangle = 0\right\}.$$

The Levi form Λ is the Hermitian form on $T^{10}(bd\ U)$

5. $$\Lambda(L, L') = \langle \partial\bar{\partial}r, L \wedge \bar{L}'\rangle.$$

U is pseudoconvex if $\Lambda(L, L)$ is a non-negative function on $bd\ U$ for all L in $T^{10}(bd\ U)$, and strongly pseudoconvex if this function is strictly positive where $L \neq 0$.

Suppose U is strongly pseudoconvex. It is a deep theorem of Fefferman (See Ref. 3) that there are smooth functions f, g so $K(z, z) = f(z)\ (-r(z))^{-n-1} + g(z)\log(-r(z))$ near $bd\ U$. In another direction, Kerzman (Ref. 5) has proved that $K(z, w)$ is c^∞ off the boundary diagonal when U is strongly pseudoconvex. This result extends to the weakly pseudoconvex case whenever the inverse to the complex Laplacian is pseudo-local. See Ref. 2. Pseudo-locality for this operator holds in the cases we consider below. It is important to understand what happens to Fefferman's theorem in the weakly pseudoconvex case. The model for the strongly pseudoconvex case is the unit ball, where $r(z) = \sum |z_i|^2 - 1$.

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