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A NEW CRITERION FOR SCATTERING THEORY

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1. Introduction.

A fundamental theorem due to Cook [1] states roughly that if H, H_0 are self adjoint operators on a Hilbert space $\mathscr{H}$ and $\Psi \in \mathscr{H}$ is such that $e^{-itH_0}\Psi \in D(H_0) \cap D(H)$ for each $t > a$ and

$$\int_a^\infty \|(H - H_0)\, e^{-itH_0}\Psi\|\, dt < \infty, \tag{1.1}$$

then the limit

$$W_+\Psi = \lim_{t \to \infty} e^{itH}e^{-itH_0}\Psi \tag{1.2}$$

exists (cf. also Jauch [2], Kuroda [3] and Kato [4]). Since that time this criterion has remained the simplest and best way of showing the existence of wave operators. For instance in the case of the Schrödinger operator

$$H_0 = -\Delta,\ H = H_0 + V,\ V = V(x), \tag{1.3}$$

Kuroda showed that the limits (1.2) exist for each $\Psi \in L^2$ if

$$(1 + |x|)^\alpha V(x) \in L^2 \tag{1.4}$$

for some $\alpha > 1 - \frac{1}{2}n$. Until the present time this has remained the weakest condition on $V(x)$ to insure the existence of the wave operators (1.2). (It is the weakest in the sense of behavior at infinity if one does not take oscillation into account).

In the present paper we give a criterion which generalizes that of Cook. Roughly speaking, our criterion states that if there are operators A, B such that

$$(Hu, v) = (u, H_0v) + (Bu, Av),\ u \in D(H),\ v \in D(H_0), \tag{1.5}$$

and

$$\int_a^\infty \|A\, e^{-itH_0}\Psi\|\, dt < \infty, \tag{1.6}$$

then the limit (1.2) exists. Applied to the Schrödinger operator (1.3), our criterion gives the existence of the limits (1.2) for every $\Psi \in L^2$ if

$$\int_a^\infty t^{-n/2}\left(\int |V(x)|^p \exp\left\{-\frac{|x - y|^2}{2(1 + t^2)}\right\}dx\right)^{1/2} dt < \infty \tag{1.7}$$

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