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RESTRICTIONS OF FOURIER TRANSFORMS TO QUADRATIC SURFACES AND DECAY OF SOLUTIONS OF WAVE EQUATIONS

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§1. Introduction

Let S be a subset of $\mathbb{R}^n$ and $d\mu$ a positive measure supported on S and of temperate growth at infinity. We consider the following two problems:

Problem A. For which values of p, $1 \leq p < 2$, is it true that $f \in L^p(\mathbb{R}^n)$ implies $\hat{f}$ has a well-defined restriction to S in $L^2(d\mu)$ with

$$\left(\int |\hat{f}|^2 d\mu\right)^{1/2} \leq c_p \|f\|_p? \tag{1.1}$$

Problem B. For which values of q, $2 < q \leq \infty$, is it true that the tempered distribution $F d\mu$ for each $F \in L^2(d\mu)$ has Fourier transform in $L^q(\mathbb{R}^n)$ with

$$\|(F d\mu)^{\wedge}\|_q \leq c_q \left(\int |F|^2 d\mu\right)^{1/2}? \tag{1.2}$$

A simple duality argument shows these two problems are completely equivalent if p and q are dual indices, $(1/p) + (1/q) = 1$. Interest in Problem A when S is a sphere stems from the work of C. Fefferman [3], and in this case the answer is known (see [11]). Interest in Problem B was recently signalled by I. Segal [6] who studied the special case $S = \{(x, y) \in \mathbb{R}^2 : y^2 - x^2 = 1\}$ and gave the interpretation of the answer as a space-time decay for solutions of the Klein-Gordon equation with finite relativistic-invariant norm.

In this paper we give a complete solution when S is a quadratic surface given by

$$S = \{x \in \mathbb{R}^n : R(x) = r\} \tag{1.3}$$

where $R(x)$ is a polynomial of degree two with real coefficients and r is a real constant. To avoid triviality we assume R is not a function of fewer than n variables, so that aside from isolated points S is a $n - 1$-dimensional C^∞ manifold. There is a canonical measure $d\mu$ associated to the function R (not intrinsic to the surface S, however) given by

$$d\mu = \frac{dx_1 \cdots dx_{n-1}}{|\partial R/\partial x_n|} \tag{1.4}$$

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