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ESTIMATES FOR THE BERGMAN AND SZEGÖ PROJECTIONS ON STRONGLY PSEUDO-CONVEX DOMAINS

D. H. PHONG and E. M. STEIN

1. Introduction

Let Ω be a bounded domain in $\mathbb{C}^n (n \geq 2)$. The Bergman operator $K : L^2(\Omega) \rightarrow L^2(\Omega)$ is the orthogonal projection of $L^2(\Omega)$ onto the closed subspace $\mathscr{H}$ of L^2 holomorphic functions on Ω; it is well-known that it can be represented as an integral operator

$$Ku(z) = \int_{\Omega} K(z, w)u(w)dw, \qquad u \in L^2(\Omega) \tag{1}$$

whose kernel $K(z, w)$—the Bergman kernel—is for each fixed z an L^2 function of w so that the above integral is well-defined. Similarly, when $b\Omega$ is smooth, the space $\mathscr{H}_b$ of L^2 boundary values of holomorphic functions is closed in $L^2(b\Omega)$, and the orthogonal projection K_b on $\mathscr{H}_b$, called the Szegö operator, can also be expressed as an integral operator with a certain kernel $K_b(z, w)$, the Szegö kernel:

$$K_bu(z) = \int_{b\Omega} K_b(z, w)u(w)d\sigma(w), \qquad u \in L^2(b\Omega) \tag{2}.$$

That K and K_b are bounded in L^2 is of course an immediate consequence of their definitions; however, to study the L^p and Hölder boundedness as well as boundedness in Sobolev spaces of higher order requires a rather precise knowledge of the singularities of $K(z, w)$ and $K_b(z, w)$ which can be obtained only under extra assumptions on Ω. When Ω is strongly pseudo-convex, a complete description has been given by Fefferman [3] and Boutet de Monvel-Sjöstrand [2]; in this paper we shall use this information to get several kinds of estimates for K and K_b in both isotropic and non-isotropic contexts, deduce some properties of holomorphic functions in several variables, and some refinements of estimates of Kohn's solution of $\bar{\partial} u = \psi$. Our results are in part extensions of earlier results and techniques of Stein [11], Folland-Stein [4], Greiner-Stein [6], and Rothschild-Stein [10]. The L^p boundedness of the Bergman operator (the case $k = 0$, of theorem 2 below) has since been generalized in a different direction, when Ω is the unit ball, by Forelli and Rudin [5]. The Λ_α parts of theorems 1 and 3 have also been obtained recently by Ahern and Schneider [1].

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