Vol. 44, No. 3 DUKE MATHEMATICAL JOURNAL© September 1977

NORMAL MAPS, COVERING SPACES, AND QUADRATIC FUNCTIONS

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1. Introduction

1.1. *Survey of results*

In this paper we investigate the relations between normal maps, covering spaces, and quadratic functions.

If $\pi : M' \to M$ is an m-fold covering and M', M are closed manifolds, then π can be interpreted as a normal map of degree m. Motivated by the theory of degree 1 normal maps, we are led to study relations between quadratic functions defined on an appropriate cohomology group of M and quadratic functions defined on the corresponding groups for M'. The theory we develop holds in the generality of coverings $\pi : X' \to X$ of Poincaré duality spaces, and using the transfer $\tau : H^*(X', \mathbb{Z}/2) \to H^*(X, \mathbb{Z}/2)$ we obtain formulas relating these associated quadratic functions.

The theory is applied to two types of problems. First we consider coverings of odd degree. Here τ is surjective and K = kernel (τ), which is analogous to a surgery kernel, inherits a canonical quadratic function $\tilde{q} : K \to \mathbb{Z}/2$. If $X' \to X$ is a principal G-bundle, $|G|$ odd, we prove that the Arf invariant of $(K, \tilde{q})$ is $\chi(X)$ if $|G| \equiv 3, 5(8)$ and 0 otherwise, where $\chi(X)$ is the (mod 2) Euler characteristic of X.

Second, if $X' \to X$ is a double cover, we construct canonical quadratic functions $\tilde{q} : H^*(X') \to Q/\mathbb{Z}$. If X is $2n$ dimensional, $H^*(X')$ means $H^n(X', \mathbb{Z}/2)$, while if X is $4n - 1$ dimensional $H^*(X')$ is the torsion subgroup, $T^{2n}(X') \subset H^{2n}(X', \mathbb{Z})$. In the $2n$-dimensional case $A[H^n(X', \mathbb{Z}/2), \tilde{q}]$ is an obstruction to a certain transversality problem for P.D. spaces. In the $4n - 1$ dimensional case our results extend the surgery product formulas of [16], [17] to P.D. spaces.

Since the first version of the present paper appeared, there have been additional applications.

First I. Hambleton and Milgram [22] have constructed free involutions s on spaces homotopy equivalent to $S^2 \times S^2$, $S^3 \times S^3$ with

$$A[H^n(S^n \times S^n, \mathbb{Z}/2, \tilde{q}] \neq 0,$$

and on taking products with CP^{2m} further examples in all even dimensions. In these examples the orbit spaces $X = X'/s$ are thus Poincaré duality spaces such that the map $f : X \to RP^N$ classifying the covering

Received February 14, 1977