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PSEUDOCONVEX DOMAINS: EXISTENCE OF STEIN NEIGHBORHOODS

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Introduction

In [3] the authors have shown that there are pseudoconvex domains $\Omega \Subset \mathbb{C}^n$ with smooth boundary such that $\bar{\Omega}$ does not have a Stein neighborhood basis. Therefore, one has to ask for additional conditions, which are weaker than strict pseudoconvexity, but, nevertheless, guarantee the existence of such a neighborhood basis. In [3], theorem 8, one such condition was already given. The purpose of this paper is to characterize another large class of weakly pseudoconvex domains Ω, the so-called regular pseudoconvex domains, for which $\bar{\Omega}$ has a neighborhood basis consisting of (strictly) pseudoconvex domains. It will turn out that the construction of the neighborhoods also works for regular pseudoconvex domains Ω lying relatively compact on an arbitrary open complex manifold, thereby giving at the same time a purely geometric criterion for holomorphic convexity in open complex manifolds.

In [2], theorem I, it was shown that on any relatively compact pseudoconvex domain Ω with smooth boundary on a Stein manifold X there is a strictly plurisubharmonic bounded exhaustion function which is Hölder continuous on $\bar{\Omega}$. In [3], theorem 6, it was shown that the corresponding Hölder exponent must be sometimes very close to 0. The construction of this paper will, however, show, that for regular pseudoconvex domains on Stein manifolds this Hölder exponent can always be chosen arbitrarily close to 1. The idea of the construction in case of a bounded pseudoconvex domain Ω with smooth boundary in $\mathbb{C}^n$ and the set-up of the paper are as follows: If $\bar{\Omega} \subset \hat{\Omega}$ and $b\hat{\Omega}$ is close to $b\Omega$, then $b\hat{\Omega}$ can be described by the positive smooth function f on $b\Omega$, which says, how far one has to go out from $p \in b\Omega$ along the exterior unit normal $n(p)$ on $b\hat{\Omega}$ at p until one hits $b\hat{\Omega}$. Therefore, the strict pseudoconvexity of $b\hat{\Omega}$ becomes a certain condition on the function f. Unfortunately, this condition seems to be too complicated for a construction of f. But the situation changes totally, if one looks for a function f, such that all the surfaces

$$b\Omega_{f,\epsilon} = \{p + \epsilon f(p)n(p) | p \in b\Omega\}$$

for $\epsilon > 0$ small enough are strictly pseudoconvex. The condition on f for this to hold is computed in §1. In §2 suppositions on Ω are given allowing the construction of such a function f. And in §3 it is shown that bounded pseudoconvex

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