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EXPLICIT DETERMINATION OF AREA MINIMIZING HYPERSURFACES

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1. Introduction

In this paper we give an explicit method for approximating the codimension one oriented surface in $\mathbb{R}^n$ with least area having a prescribed boundary, provided there is but one hypersurface of least area with that boundary. We do not restrict, beforehand, the topological type of the desired surface nor the singularities it may have. It should be noted that the uniqueness criterion above is satisfied for a dense set of boundaries.

The problem of finding a surface of least area with given boundary was first investigated by Lagrange in [L; Appendix I] where he introduced the minimal surface equation. Subsequently, the problem was related to the vanishing of the mean curvature of the surface by Meusnier (see [DG; §176]) and specific examples of minimal surfaces were obtained by Meusnier, Scherk, Enneper and others (see [DG; §§176, 181, 207]). In modern times, various existence theorems have been proved (for example, in [FF; §9]) which assure us that the problem of finding a surface of least area with given boundary has a solution in all dimensions and codimensions. Examples due to Fleming, [FW1], and Bombieri, De Giorgi, and Giusti, [BDG], show that in seeking an oriented surface of least area one cannot restrict a priori the topological type of the surface nor, in higher dimensions, refuse to allow singularities.

The problem of approximating, by computational means, the surfaces of least area which the existence theorems provide has been little studied ([NJ; VI.3]). Douglas's work in [DJ] is perhaps the first attempt. It deals with the non-parametric problem over a two-dimensional domain without any consideration of convergence questions. More recently, the non-parametric problem has been treated in this context with proofs of convergence in [JC] and [PH]. The latter paper deals with the problem over domains in $\mathbb{R}^n$ ($n \geq 2$) and includes an a priori estimate of the maximum difference of the approximation and the true solution.

Approximation of parametric surfaces of least area has, to our knowledge, only been treated in the two-dimensional case for surfaces with the topological type of a disc without any proof of convergence (in [WW]). The method of [WW] cannot be generalized to higher dimensions because of its reliance on isothermal parameters.

The method of this paper is to reduce the parametric problem in $\mathbb{R}^n$ to a non-

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