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SUBMANIFOLDS OF CONSTANT POSITIVE CURVATURE I

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Dedicated to Herbert Potter Moore

1. Introduction. This article is concerned with the structure of isometric immersions from the n-dimensional sphere S^n of constant curvature one into N-dimensional Euclidean space E^N, when $N \leq 2n - 1$. The approach is based upon an algebraic investigation of the second fundamental form of a developable submanifold in a pseudo-Riemannian space form; we are thus led to the purely algebraic theory of "flat bilinear forms." Using this theory we will show, for example, that the three-dimensional constant curvature real projective space P^3 has no isometric immersion in E^5.

The theory of flat bilinear forms possesses additional applications beyond the theory of constant curvature submanifolds. For example, the theory of flat bilinear forms sheds light on the structure of conformally flat submanifolds of E^N [21]. It is to be hoped that it will also yield insight into the "rigidity problem" for more general submanifolds. Thus the theory of flat bilinear forms (§§4–6) will be developed in somewhat greater generality than is needed for the application to constant curvature submanifolds.

Understanding the structure of isometric immersions $S^n \to E^N$ requires two steps. First one must classify the various possible *local* solutions to the isometric immersion problem; then one must determine how the various local solutions piece together to form the *global* isometric immersion. The first step requires a solution to the purely algebraic problem of classifying the second fundamental forms which satisfy the Gauss equation (at a single point). When $N \leq 2n - 2$, there is only one possible structure for the second fundamental form of an isometric immersion from an open subset U of S^n to E^N. This fact was discovered by O'Neill [22]. In this article, we will show that when $N = 2n - 1$, there are exactly two possibilities for the structure of the second fundamental form (see §3), and there are two corresponding types of local isometric immersions.

It is enlightening to specialize to the case ($n = 2$) of surfaces of constant curvature one in E^3. In this case the two possible structures for the second fundamental form at a point are easily described: the point can be either umbilic or nonumbilic. In the general case of isometric immersions $S^n \supset U \to E^{2n-1}$ there is a similar dichotomy: there are two types of points, corresponding to the two possible structures of the second fundamental form, which we call

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