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MORAVA STABILIZER ALGEBRAS AND THE LOCALIZATION OF NOVIKOV'S E_2-TERM

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The E_2-term of the Adams–Novikov spectral sequence [2] for a spectrum X localized at the prime p has the form

$$\mathrm{Ext}_{BP_*BP}{}^* (BP_* , M) \tag{0.1}$$

where BP is the Brown–Peterson spectrum [2] at p and M is the "BP_*BP-comodule" [1] $BP_*(X)$. Recall [2] that $BP_* = \pi_*(BP) = \mathbf{Z}_{(p)}[v_1 , v_2 , \cdots]$, $|v_i| = 2p^i - 2$. The purpose of this paper is to identify (0.1) with an Ext group over a smaller "Hopf algebra" in case M is *v_n-local,* by which we mean that v_n acts on M bijectively.

The first theorem in this direction is due to Jack Morava [14]. Morava shows that if M is a v_n-local comodule which is killed by the ideal $I_n = (p, v_1 , \cdots , v_{n-1})$ and finitely generated over $v_n^{-1}BP_*/I_n$, then (0.1) may be computed in terms of the continuous cohomology of a certain p-adic Lie group with coefficients in a finite dimensional representation over $\mathbf{F}_{p^n}$ constructed out of M.

We prove the following "covariant" analogue of this theorem in Section 2. Let $K(0)_* = \mathbf{Q}$, and $K(n)_* = \mathbf{F}_p[v_n , v_n^{-1}]$ for $n > 0$, with the obvious BP_*-algebra structures. Let $K(n)_*K(n) = K(n)_* \otimes_{BP_*} BP_*BP \otimes_{BP_*} K(n)_*$; it inherits from BP_*BP the structure of a Hopf algebra over the graded field $K(n)_*$.

THEOREM 2.10. *If M is v_n-local and $I_nM = 0$, then*

$$\mathrm{Ext}_{BP_*BP}{}^* (BP_* , M) \cong \mathrm{Ext}_{K(n)_*K(n)}{}^*(K(n)_* , K(n)_* \otimes_{BP_*} M)$$

under the natural map.

In Section 3 we strengthen Theorem 2.10 by dropping the requirement that $I_nM = 0$. Let $E(n)_* = \mathbf{Z}_{(p)}[v_1 , \cdots , v_n , v_n^{-1}]$ with the obvious BP_*-algebra structure, and let $E(n)_*E(n) = E(n)_* \otimes_{BP_*} BP_*BP \otimes_{BP_*} E(n)_*$. Then we have

THEOREM 3.10. *If M is v_n-local, then*

$$\mathrm{Ext}_{BP_*BP}{}^* (BP_* , M) \cong \mathrm{Ext}_{E(n)_*E(n)}{}^* (E(n)_* , E(n)_* \otimes_{BP_*} M)$$

under the natural map.

Thus higher generators can be neglected, at the cost of introducing a rather complicated set of relations into BP_*BP.

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