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DISTINGUISHED CURVES IN PSEUDOCONVEX BOUNDARIES

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Introduction.

If M is a real hypersurface in $\mathbf{C}^n$ through the origin, with non-degenerate Levi-form, then in [3] Chern, following E. Cartan, associates to every tangent vector $\xi \in T_o(M)$, transverse to the maximal complex subspace $H_o(M)$, a unique curve γ in M, called a *chain*. These curves are CR-invariants on M, and one hopes they will play a role in the geometry of hypersurfaces similar to that of geodesics in Riemannian geometry. In particular, Moser has shown [3] that (in the case of real analytic M with definite Levi form) one may osculate M to maximal order by the holomorphic image of the unit sphere in $\mathbf{C}^2$ along a chain γ.

In [4], C. Fefferman introduced another family of curves on M by projecting light rays (null-geodesics) from a circle-bundle over M with indefinite metric. These curves are CR-invariant as well, each determined by a $\xi \in T_o(M) - H_o(M)$ as above. The conformal class of the metric is also invariant under biholomorphic maps. In [4], Fefferman demonstrates some striking behavior for these projected curves on M, taking advantage of the Hamiltonian form of the geodesic equations, and a relatively simple algorithm for computing the metric, given a defining equation $\psi = 0$ for M.

In this paper, we present an intrinsic construction of the circle bundle and the conformal class of metrics for an integrable CR-manifold. We build these directly out of Chern's structure bundle Y and its Cartan connection ω. We then construct the classical conformal structure bundle P for this class of metrics and give a natural imbedding of Y into P under which the canonical Cartan connection $\tilde{\omega}$ on P pulls back to ω on Y. Motivated by the Cartan–Chern definition of chains, we define invariant differential systems on P which, given the comparison of $\tilde{\omega}$ and ω above, clearly restrict to the systems on Y defining chains. These systems on P are readily interpreted, and in particular, the curves they define on the circle bundle are light rays.

Finally, for $M \subset \mathbf{C}^n$ as in the beginning of the introduction, we compute our metric, and show that it is given by Fefferman's formula. In particular, Fefferman's curves on M are chains. However, the imbedding of Y into P, with the comparison of ω and $\tilde{\omega}$, shows that all CR-invariants of M are com-

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