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FACTORIZATION THEOREMS FOR FUNCTIONS IN THE BERGMAN SPACES

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1. For $0 < p < \infty$ and $0 \leq \alpha < \infty$, we consider the Bergman spaces $A^{p,\alpha}$ of functions analytic in the unit disc U of $\mathbf{C}$ and satisfying

$$||f||_{p,\alpha}{}^p = \iint_U |f(z)|^p (1 - |z|^2)^\alpha (dA(z) < \infty,$$

where dA denotes the normalized Lebesgue measure $(1/\pi)\ dxdy$. The spaces $A^{p,0}$ are denoted simply A^p.

In the present paper, we discuss factorization properties of functions in these spaces. Our first main result is the following:

THEOREM 1. *Let p and α be fixed, and let $n \geq 2$ be an integer. Then there is a constant C depending only on p, α, and n, such that if $f \in A^{p,\alpha}$ there exist functions $f_1 \cdots f_n \in A^{pn,\alpha}$ with*

$$f = \prod_{i=1}^n f_i$$

and

$$\prod_{i=1}^n ||f_i||_{pn,\alpha} \leq C\ ||f||_{p,\alpha}\ .$$

Theorem 1 has a somewhat interesting history. The analogous theorem for H^p spaces over U in place of $A^{p,\alpha}$ is classical. Rudin [9] showed that this result does not generalize to H^p spaces over a polydisc, at least in dimensions greater than 3. Somewhat later, Miles [7] and Rosay [8] extended Rudin's counter-example to dimensions 2 and 3. Interest in the present problem arose partly because of connections between A^p spaces and H^p spaces in several variables. Indeed, in view of the theorem in [5], a negative result in place of Theorem 1 would have reproved the result of Rudin, Miles and Rosay. In the other direction, recent work of Coifman, Rochberg, and Weiss [2] on H^p spaces in several variables has yielded as a corollary a weaker version of Theorem 1.

Our second main theorem deals with division by Blaschke products, which products we define here mainly for purposes of fixing notation. For $a \in U$, we define first the Möbius transformations

$$C_a(z) = \frac{a - z}{1 - \bar{a}z} \tag{1.1}$$

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