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ON A CERTAIN CLASS OF FUCHSIAN PARTIAL DIFFERENTIAL EQUATIONS

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§1. Introduction.

Let X be an n dimensional manifold and $P : C^\infty(X) \to C^\infty(X)$ a pseudo-differential operator with real principal symbol. Let H_p be the Hamiltonian vector field

$$H_p = \sum \frac{\partial p}{\partial \xi_i} \frac{\partial}{\partial x_i} - \frac{\partial p}{\partial x_i} \frac{\partial}{\partial \xi_i}. \tag{1.1}$$

Hörmander and Duistermaat have shown that if H_p isn't parallel to the radial vector field

$$\Xi = \sum \xi_i \frac{\partial}{\partial \xi_i} \tag{1.2}$$

at the point (x, ξ) then in a neighborhood of (x, ξ), P can be conjugated by a Fourier integral operator to the simple form

$$-i \frac{\partial}{\partial x_n} \quad \text{where} \quad i = \sqrt{1}.$$

Using this theorem they are able to prove the following two results about the homogeneous equation $Pu = 0$. 1) If u is a solution of this equation and $(x, \xi) \in WF(u)$ then the bicharacteristic through (x, ξ) is in $WF(u)$. 2) There exist solutions of this equation, at least locally, with singularities concentrated along single bicharacteristics.

In this paper we obtain some analogous results for the case when H_p is parallel to the radial field at (x_0, ξ_0). Such a point will be called a radial point. If

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